Fifty Years of Condensed Matter Physics:
A Symposium on the occasion of Vinay Ambegaokar's retirement
Saturday, June 16, 2007
Schwartz Auditoruim, Rockefeller Hall

Sponsored by the College of Arts & Sciences and the Department of Physics

8:30 Opening Remarks: Peter Lepage

Chair: Bruce Patton

8:40 Allan Griffin, University of Toronto
Many body physics in the 1960s: A Golden Age

9:05 Anthony Leggett, University of Illinois
Quantum dissipation in the 80s and today

9:35 Jason Ho, The Ohio State University
New challenges and future opportunities in cold atoms

10:05 Break

Chair: Alexei Maradudin

10:30 David Mermin, Cornell University
Things I wish they had told me about Shor's factoring algorithm

11:00 Eric Siggia, Rockefeller University and Cornell University
Unconventional physics problems from biology

11:30 Alexis Baratoff, University of Basel
Reduction of friction in atomic-scale contacts — A novel model

12:00 Lunch

Chair: Joseph Serene

1:30 Gerd Schön, University of Karlsruhe
Solid-state qubits and decoherence

2:00 Mohit Randeria, The Ohio State University
High T_c superconductivity: Where are we after 20 years?

2:30 Sudip Chakravarty, University of California, Los Angeles
Entanglement

3:00 Break

Chair: Aashish Clerk

3:30 Jan Von Delft, University of Munich
Mesoscopic to universal crossover of transmission phase of multi-level quantum dots

4:00 Bertrand Halperin, Harvard University
Number parity in superconductors and the f=5/2 fractional quantized Hall state

4:35 Walter Kohn, University of California, Santa Barbara
The Power of the Sun (a movie about 1 hour long)

6:30 Dinner



10:05 Coffee/tea break, hall outside of Schwartz Auditorium. Please remember that no food or drink is allowed in Schwartz.

12:00 Lunch catered by Hope's Way, 700 Clark.

3:00 Coffee/tea break, in hall outside of Schwarz

6:30 Dinner catered by Hope's Way, 700 Clark

MC/remarks: Jim Langer

Lunch, $12.50
Dinner, $50.00, includes wine and champagne

Registration: http://www.lassp.cornell.edu/lassp_data/LASSP/VA-Registration.html

Many body physics in the 1960s : A golden age

Allan Griffin (Ph.D., Cornell, 1965)
University of Toronto

Symposium : Fifty years of condensed matter physics

Fifty year ago, in 1957, two special things happened:

1. Vinay started his Ph. D. research.

2. The BCS long paper was published.

The anniversary of the famous BCS paper reminded me that in many ways, **1957 was an absolutely pivotal year in the history of of how we think about many body systems in condensed matter physics.** This will be the subject of my talk today on this special occasion. Vinay said that he wanted us to talk about current research, not the **old days**. Since I took mandatory retirement in 2004, I feel that I am now his **senior** and can go against the wishes of my supervisor.

In a nutshell, I will argue that **in the years 1957-1960, the way we think about interacting many body systems underwent a revolution.** In theoretical physics, it compares with the the **development of quantum mechanics** in the golden period of 1925-1927.

The **magic year** was precisely 50 years ago, in 1957. A almost endless number of papers were published in a single year that, looking back, **lead to a paradigm shift**. Many classic problems that had been nagging theorists since the 1930s were completely solved. Perhaps more important , we learned new words and a new language to describe interacting systems.

We developed a new set of criteria that defined what we meant when we said we understood a phase of condensed matter.

Condensed matter theory in the period 1950-1956

Nagging problems in solid state physics:

1. Theory of metals.
A good description was based on a Fermi gas of independent electrons moving in the periodic lattice of ions.This was the famous **Sommerfeld-Bloch picture developed around 1930.** Why could one forget about the strong Coulomb interactions between electrons? More precisely, why could we neglect the so called **correlation energy?**

2. Theory of superconductivity.
There were a lot of useful **phenomenological theories**, such as the **Ginzburg-Landau(GL)** theory(1950) and the **London** equations (1935). But there was no underlying **microscopic basis** for these theories , or any idea of the cause of this strange **phase transition**.

What did theorists do in the period 1950-1956 ?

They knew that one had to include **interactions** somehow.The main approach was to introduce some correlations into the many particle wave function for the ground state of the system, and treat it **variationally**.

This precluded any ability to deal with a **phase transition**, since the concept of an **order parameter** was still not really understood . Moreover it was difficult to build an order parameter into a quantum mechanical wavefunction!

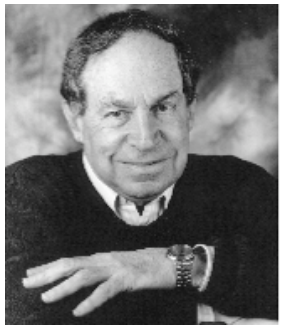
The greatest success of this approach was with **interacting Bosons**. **Feynmann** (1954) intoduced a variational groundstate wavefunction for a Bose liquid. In 1956, **Oliver Penrose** and **Onsage**r used this kind of wavefunction to define what a Bose condensed phase was and estimated the condensate fraction in superfluid He^4 to be 8%.

Bohm and Pines- the breakthrough in the early 1950s

David Bohm and his graduate student David Pines tried to develop a theory of the collective modes in an interacting electron gas. Their approach was to introduce the coordinates which would describe the these collective degrees of freedom into the quantum mechanical equations of motion describing density and velocity variables.

In a series of seminal papers (1951-1953), Bohm and Pines developed what we now call the RPA theory of plasmon oscillations in metals, separate from the single particle excitations which now interacted by a weak screened Coulomb interaction. However, the Bohm-Pines theory was controversial, because it was felt that there was an overcounting problem with the degrees of freedom. The challenge was to justify their theory.





The dense electron gas problem solved in 1957!

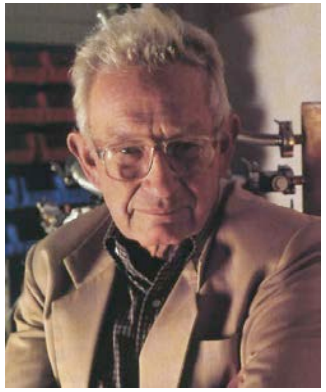
The Bohm-Pines RPA theory became a challenge to theorists. The problem was solved by the work of many people, but especially in a paper by **Gell-Mann** and **Brueckner** published the magic year of 1957. They summed up the leading order Feynmann diagrams in a dense electron gas and evaluated the correlation energy in the ground state, getting agreement with the Bohm-Pines results.

Within months, the explicit role of the **plasmons vs particle-hole excitations** was elucidated by GB working with **Robert Brout** and **Sawada**.

This success made **diagrammatic methods and Green's functions** very popular, as did a flood of papers in 1957 using and developing these methods.

Papers on interacting electron systems in 1957

Landau (1956 &1957)
Beliaev
Galitskii
Migdal
Hubbard
Nozieres
Gell-Mann & Brueckner
Anderson
Brout
Bogoliubov







Goldstone
Thouless
Hugenholtz

. . . .

A new language developed after 1957

The structure of correlation functions in space and time
Single particle Green's functions
Linear response functions and Kubo formulas
Order parameters
Broken symmetry and Goldstone modes
Renormalized interactions and vertex functions
Propagator renormalization
Collective modes vs single particle excitations
Collisionless vs hydrodynamic regions

The age of theoretical summer schools

This new gospel was transmitted at many summer schools on the many body problem in the following years, starting with Les Houches in 1958 (Nozieres). In the 1960s, Vinay was a frequent lecturer on Fermi liquid and BCS theory.

Normal Fermi Liquid Theory-1957

In 1957, Landau formulated a theory of an interacting Fermion gas, based on the assumption that a **sharp Fermi surface** well known in non-interacting Fermi gases would **persist** even in the presence of strong interactions. Finally we had a **definition** of what a **metal** was! Now called a **normal Fermi liqiud.** However, it was a phenomenological theory. It stimulated a whole series of papers in the next few years with the aim to give a precise **microscopic basis of Landau theory** using Green's functions and many body theory. Key papers were by **Kohn , Luttinger and Ward,** as well as by **Nozieres**.

Landau





Nozieres

BCS theory of Superconductivity-1957

Bardeen, Cooper and Schrieffer finally solved the mystery of superconductivity based on the idea of Cooper pairs. Their theory was magnificent physics but "old fashioned" in the sense that it was based on an explicit many particle ground state wavefunction and excited states.

Within a few months, Gorkov showed that the BCS theory corresponded to a simple mean field theory but besides the the usual Hartree-Fock terms, one had to also include a off-diagonal mean field describing the effect of " a sort of Bose condensate of Cooper pairs" (to quote Gorkov)

It is curious that it would only be in 1985 that Leggett re-emphasized that BCS should be thought in terms of such a Bose condensate of bound pairs

The key new concept introduced by Gorkov introduced was an anomalous off-diagonal single particle propagator related to bound pair formation

$$G_{12} \propto \langle \psi(r,t)\psi(r',t') \rangle$$



in addition to to normal diagonal propagator

$$G_{11} \propto \langle \psi^{+}(r,t)\psi(r',t') \rangle$$

This allowed Gorkov to re-derive the whole complex BCS theory in a 3 page paper. The condensate mean field is

$$\Delta \propto -U\langle \psi\psi \rangle$$

and gives rise to the energy gap in the BCS quasiparticle spectrum.

Landau's group in Moscow, 1956.

A. Abrikosov, I. Khalatnikov, L. Landau, E. Lifshitz.
L. Pitaevskii, I. Dzyaloshinskii.



The famous **AGD** book on was being hatched by these people!

The Martin-Schwinger school at Harvard

Another school similar to the Landau powerhouse grew out of the famous paper by **Martin and Schwinger** (1959).

A whole group of graduate students came out of Harvard in the early 1960s using the mantra of **thermal Green's at imaginary frequencies**. This work was captured best by the classic book by **Kadanoff and Baym** (1962) on Green's function techniques for non-equilibrium problems.

Vinay received his Ph.D. in 1960 and joined Cornell in 1962. He mastered the Martin-Schwinger techniques and used them on a variety of novel problems in the 1960s in the area of superconductivity.

Moscow 1965. USSR-USA Seminar.

I. Khalatnikov, L.Pitaevskii, G. Eliashberg,
A. Abrikosov, D. Pines , P. Martin,
L. Kadanoff, J. Bardeen



The Gorkov formulation of BCS led to a whole bunch of extensions to new kinds of superconductors

The subject was pounced on by the Landau school!

- **Strong - coupling theory** - Eliashberg, 1960
- **Gapless superconductivity** - Abrikosov and Gorkov,1960
- **Derivation of the GL theory of Type-II superconductors** - Gorkov, 1959

- **Josephson effect** - Josephson, Ambegaokar and Baratoff

- **Collective oscillations of the Cooper pair condensate** -Anderson(1958), Bogoliubov, Ambegaokar and Kadanoff

- **p-wave and d-wave pairing** - found many years later in superfluid He^3 and in high T_c superconductors.

Theory of Bose superfluids by Beliaev-1957

The idea was to split the quantum field operator into two parts

$$\psi = \langle \psi \rangle_{BS} + \tilde{\psi}$$

With the broken symmetry order parameter separated out, the **noncondensate part** could be treated by regular many Body diagrammatic techniques.



Beliaev's work was done **down the hall** from Gorkov, who was introducing the analogous formalism for **superconductors**.

Beliaev's approach had great success in formulating the dynamical role of a Bose condensate in a liquid, even if calculations could not be made. The **conceptual basis** was settled in the period 1957-1964, with key papers by **Gavoret** and **Nozieres** (1964) and **Hohenberg** and **Martin** (1965).

Work on the many body theory of interacting Fermi and Bose quantum gases started in 1957 is now having a revival because of experiments in ultracold atomic gases.

Many of these original calculations could never be tested until ultracold gases could be produced in the lab, starting in 1995.

The whole field of many body physics is being pushed along with these new studies.They are the the dream system of many body theorists, since the interaction strength and density can adjusted at will by turning knobs!

BCS-BEC crossover in ultracold Fermi gases

The pioneering work of BCS- Gorkov on superconductivity and Beliaev on BEC has been beautifully joined in the last few years in **two component atomic Fermi gases**.
As first studied in detail by **Leggett** in 1980, by increasing the attractive interaction between Fermions, one has a **smooth crossover** from a standard weak coupling superconducting BCS phase to an interacting gas of Bosonic molecules.

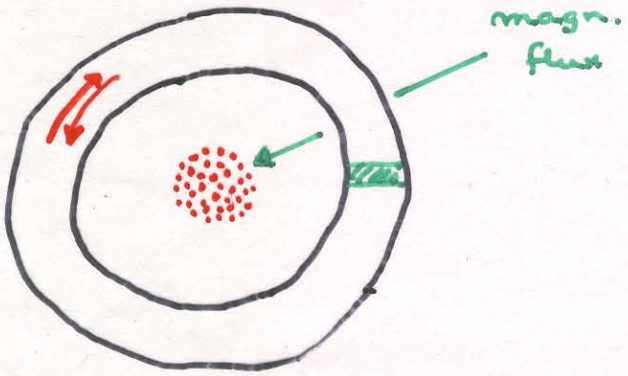
This **BCS-BEC crossover** is a hot topic in current research in ultracold atomic Fermi gases, verifying work at **finite** temperatures done many years ago by **Randeria**(1993, 1997) and **Nozieres** (1985).

A magnificent achievement

The work that solved the problem of normal metals and superconductors was a magnificent achievement, It was part of the revolution in our understanding of many body problems starting in 1957. They should be celebrated as a **high point in the history of theoretical physics**.Vinay and many of us had the privilege of building on this base.

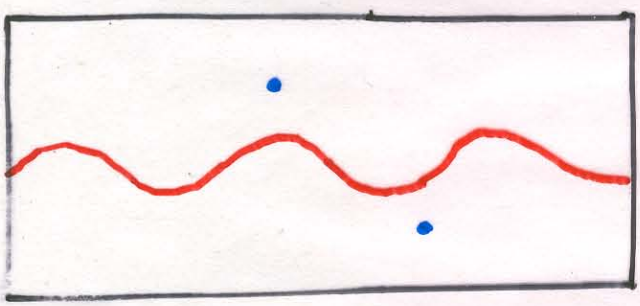
In recent years, one has a tendency of referring to normal metals and the BCS superconductors as **boring** , as we grapple with **highly-correlated metals and high temperature superconductors**. Perhaps it is a sort of jealousy, as we try to find an equivalent set of **powerful ideas** that first emerged in 1957. We should also remember that these theories gave us the language that we use now to discuss all many body systems, even these unsolved exotic phases!

Some "Hybrid" Optical-Solid State Qubits



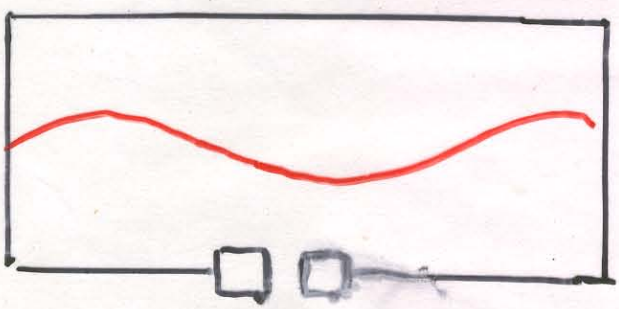
RF SQUID

(Delft, SUNY, Saclay ...)



Cavity QED

(ENS)



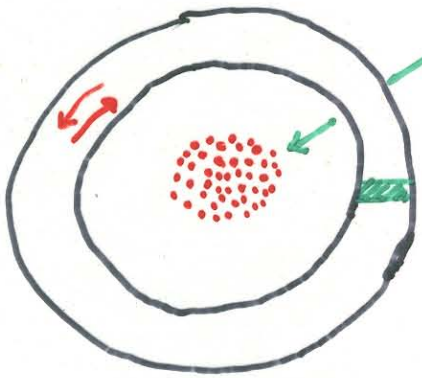
Superconducting qubit coupled to QED cavity

(Yale)



Surface plasmon

(Geneva)



electron states

$$\Psi \sim |1\rangle|\Phi_1\rangle + |2\rangle|\Phi_2\rangle$$

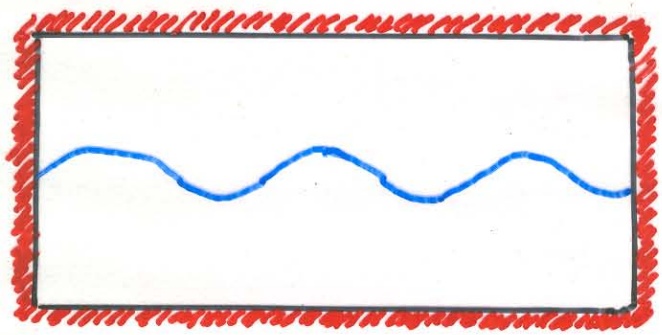
$$|1\rangle \sim (\exp i\alpha\theta)^{N/2}$$

$$|2\rangle \sim (\exp -i\alpha\theta)^{N/2}$$

← no. of Cooper pairs $\sim 10^6 - 10^9$

($\alpha \ll 1$ for Delft expt., but ~ 1 for SUNY expt.)

so $\langle 1|2\rangle \ll 1 \Rightarrow$ SS DOF's strongly (essentially) involved.



cavity QED (no atoms)

"Naive" description:

$$\Psi \sim a|\text{vac}\rangle + b|\gamma\rangle$$

← 1-photon state

but:

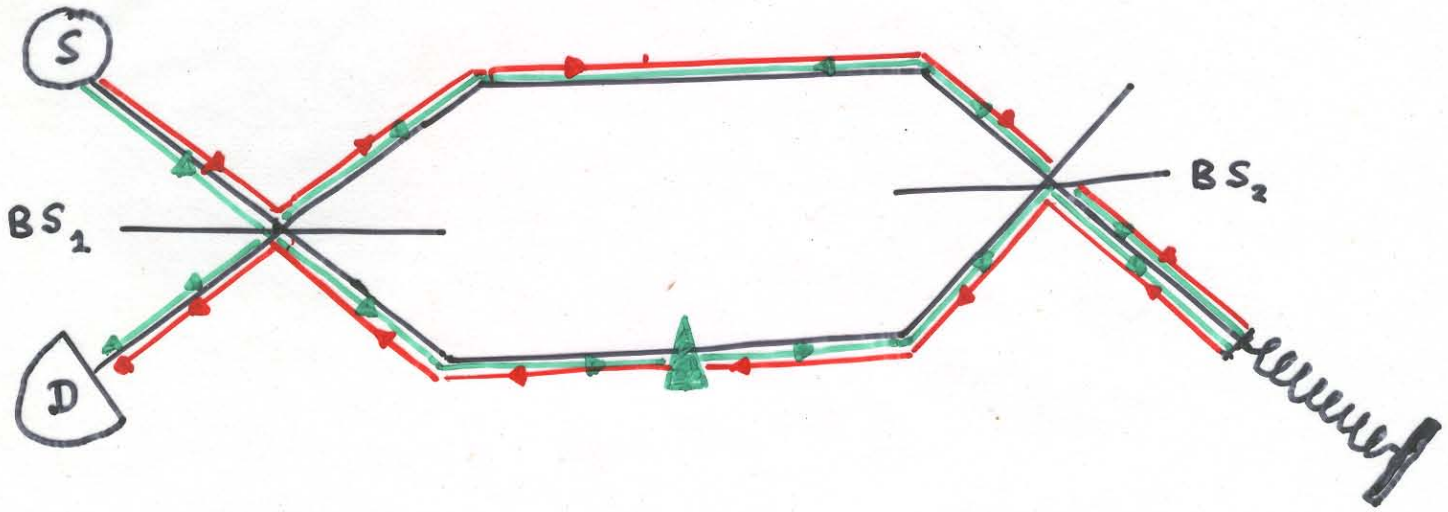
"photon only confined to cavity bec. strongly interacting with electrons in walls"

$$\Rightarrow \quad \Psi \sim a|\text{vac}\rangle|1_e\rangle + b|\gamma\rangle|2_e\rangle.$$

← states of electrons in walls

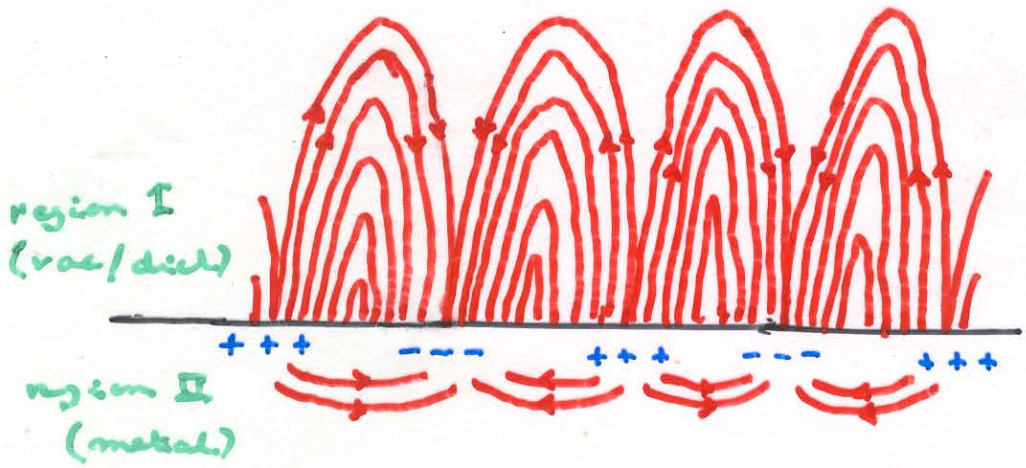
$$\langle 1_e|2_e\rangle \ll 1 \quad ??$$

S. Fasel et al., NJP 8, 1 (2006): (schematic)



(also: EPR-Bell expt. with converters for each photon)

What is a surface plasmon?



requires

$\epsilon_1 + \epsilon_2 < 0$

i.e. metal-dielectric interface

Total no. of electrons displaced into surface zone:

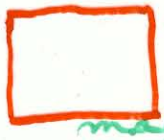
$\sim (\omega/\omega_p) \cdot (2\pi L/\lambda) \sim 300$ for Fasel et al. expt.

$\gg 1$!

- ω_p: bulk plasma freq.
- L: lateral extent of S.P. wave packet
- λ: free-space wavelength of photon

How to define "degree of involvement"?

In absence of S.P.:



$\Psi \equiv \Psi_0 = \Psi_0(1,2)$ strongly entangled

in presence of S.P.

$\Psi \equiv \Psi_\gamma = \Psi_\gamma(1,2)$ also strongly entangled.

how to compare $\hat\rho_\gamma^{(2)}$ and $\hat\rho_0^{(2)}$?

reduced DM of 2 (metal)

A first shot: use Jozsa definition of fidelity F for mixed states:

"inv."

$$J \equiv 1 - F = 1 - \left\{ \mathrm{Tr}\left(\sqrt{\hat\rho_\gamma}\, \hat\rho_0 \sqrt{\hat\rho_\gamma} \right)^{1/2} \right\}^2$$

Problem: if

$$\hat\rho_\gamma = (1-\epsilon)\hat\rho_0 + \epsilon \Delta\hat\rho \qquad (\mathrm{Tr}(\hat\rho_0 \Delta\hat\rho) = 0)$$

then $J \sim \epsilon^2$ not $\sim \epsilon$.

So: use simply

$$J = \epsilon$$

In special case

$$|\gamma\rangle = (\alpha\, \hat\Omega_1^\dagger \times \hat 1_2 + \beta\, \hat 1 \times \hat\Omega_2^\dagger)|0\rangle$$

$$\langle 0|\hat\Omega_j|0\rangle = 0, \quad \langle 0|\hat\Omega_i^\dagger \hat\Omega_j|0\rangle = \delta_{ij}$$

df. simplifies to

$$J = |\beta|^2$$

The $64K question: How "involved" are the metallic electrons? i.e. J is "share" of exciton attributable to metal.

If excitation of metal can be expressed in terms of normal modes (k,n):

↑ —— w.v. normal to surface

then

$$J = \tfrac{1}{2}\sum_{kn}\left\{|X_{kn}|^2/\langle|X_{kn}|^2\rangle_0\right\}$$

$X_{kn} \equiv$ amplitude of excit^n of mode k,n in S.P.,

$\langle|X_{kn}|^2\rangle_0 \equiv$ mean-square zero-point amplitude

A SIMPLE MODEL: "LONDON" DYNAMICS

df. of model: $\underset{\sim}{j}(\underset{\sim}{r}t) = -\epsilon_0\omega_p^2\,\underset{\sim}{A}(\underset{\sim}{r}t)$ ← vector pots!

el. current

bulk plasma freq. $\equiv (ne^2/m\epsilon_0)^{1/2}$

In this model there is only one long^l (L) and one transverse (T) mode for each k, with freq.

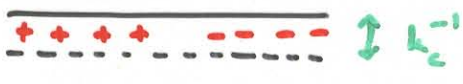
$$\omega_k = \omega_p \qquad (L)$$

$$\omega_k = (\omega_p^2 + c^2k^2)^{1/2} \qquad (T)$$

Express contrib^{ns} to J in terms of value of $A(rt)$ just inside metal in S.P., A_0:

L case:

$$J \sim \epsilon_0\omega_p\sum_k |A_k|^2 \qquad A_k \sim A_0 k_c^{-1} \text{ up to } k \sim k_c$$



so,

$$J_L \sim \frac{\epsilon_0}{2\pi\hbar}\left(\frac{\omega_p}{k_c}\right)|A_0|^2$$

Transverse contribution:

$$A_k \sim A_0 \left(K(\omega) + ik\right)^{-1}$$

$K(\omega) \equiv (\omega_p^2 - \omega^2)^{1/2}/c$

so:

$$J_T = \frac{\epsilon_0 c^2}{2\pi k} |A_0|^2 \int_0^\infty \frac{dk\, \omega(k)}{\omega^2(k) - \omega^2} \qquad (\omega < \omega_p)$$

Avoid log'c divergence (a) by cutoff at $k \sim k_c$, or (b) by noting that most of "involvement" of metal for $\omega_k \gg \omega_p$ is simply that of EM DOF: the contrib^n of electronic DOF's is only a factor $\sim \omega_p^2/\omega_k^2$. So, multiplying integrand by this factor,

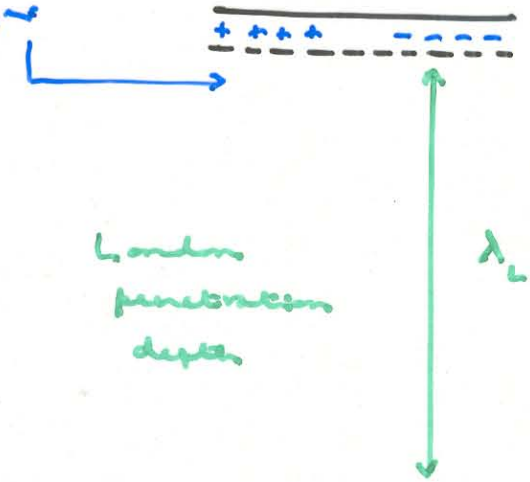
$$J_T \sim \frac{\epsilon_0 c}{\pi k} |A_0|^2 F(\omega/\omega_p)$$

~ 1 unless $\omega \sim \omega_p$

Since generally $ck_c \gg \omega_p$, $J_L \ll J_T$ and

$$\boxed{J \lesssim (\epsilon_0 c/\pi k)|A_0|^2}$$

(recall: A_0 is EM VP of SP "just inside" metal, i.e. just beyond surface layer



[Will verify below that this is correct o. of m. also for more realistic models]

General Expression for J:

[lots of messy algebra ⇒]:

$$J_L = \frac{\epsilon_0}{2\pi t}(\omega_p/k_c)|A_0|^2\,\Psi(\omega)$$

$$J_T = \frac{\epsilon_0 c}{4t}\,\Phi(\omega)$$

where dim'less functions $\Psi(\omega)$, $\Phi(\omega)$ are given by:

$$\Psi(\omega) \equiv \omega_p^{-1}\omega^4|\epsilon_L(\omega)|^2 \cdot \frac{2}{\pi}\int_0^\infty \frac{d\omega'}{(\omega'^2-\omega^2)^2}\,\mathrm{Im}\left(-\frac{1}{\epsilon_L(\omega')}\right)$$

not wrt ω!

$$\Phi(\omega) \equiv \Phi(\omega:\underline{k}_\parallel) \equiv \frac{2c}{\pi}\int_0^\infty \frac{d\omega'\,\omega'^2}{(\omega'^2-\omega^2)^2}\,\mathrm{Im}\left\{\frac{|K(\omega:\underline{k}_\parallel)|^2 - K^2(\omega':\underline{k}_\parallel)}{K(\omega':\underline{k}_\parallel)}\right\}$$

$$K(\omega:k_\parallel) \equiv c^{-1}\left(-\epsilon(\omega)\omega^2 + c^2k_\parallel^2\right)^{1/2}$$

↑

= complex inverse penetration depth of mode of freq. ω.

Note: both $\Psi(\omega)$ and $\Phi(\omega)$ are of the form

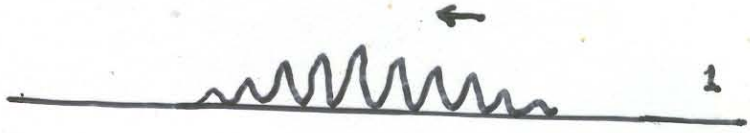
(integrand singular at $\omega'=\omega$)

$$\text{const.}\int_0^\infty \frac{F(\omega')}{(\omega'-\omega)^2}\,d\omega'$$

How to handle divergence from $\omega' \sim \omega$? Replace monochromatic wave by wave packet, s.t.

$$A_0 \equiv A_0(t) = \mathcal{A}_0 f(t).$$

$$\int_{-\infty}^{\infty}|f(t)|^2\,dt = 1.$$



For a wave packet, $A_0(t) \sim f(t)$,

$$J \equiv J(t) = J_{nw}(t) + J_{irrw}(t)$$

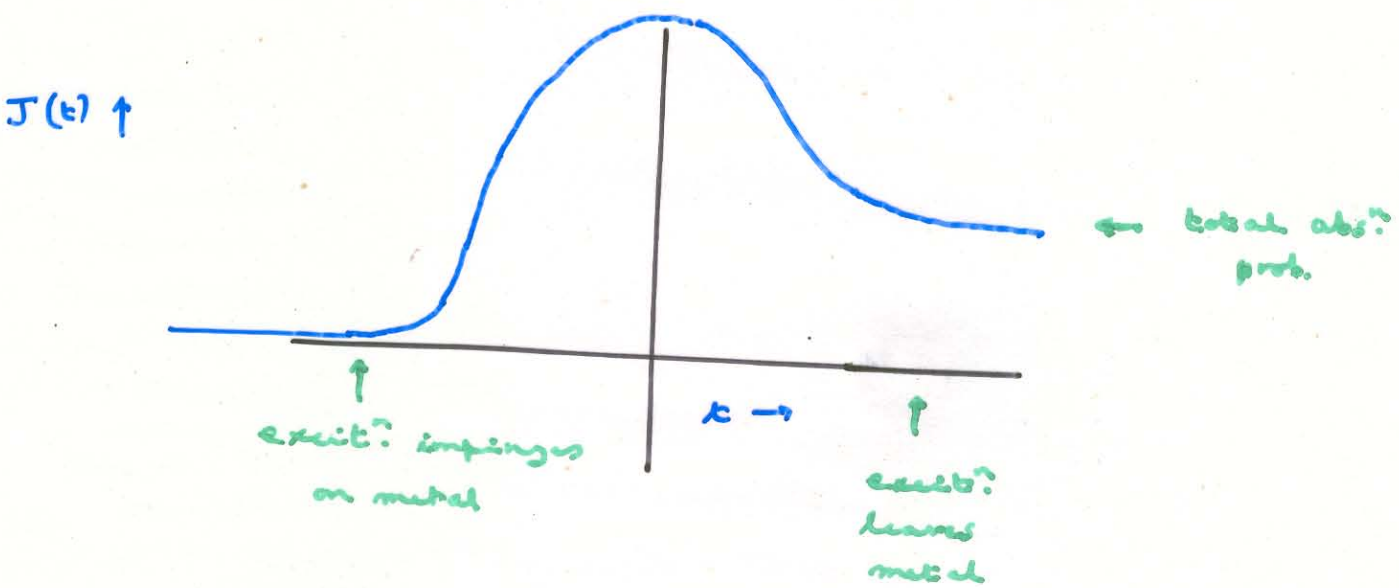
$$J_{nw}(t) = J_{ns} |f(t)|^2$$

↑ contrib. of nonsingular terms of ω'-integrand

$$J_{irrw}(t) = c\epsilon_0 |\mathcal{Z}_0|^2 \left(-\operatorname{Im} \kappa(\omega_0)\right) Q(t)$$

$$Q(t) \equiv \int_{-\infty}^{t} |f(t)|^2 \, dt$$

$\Rightarrow$ $J_{irrw}(t)$ is simply prob. that exciton has been absorbed up to time t.



What about the "irreversible" contrib.?

e.g. for long. case, must evaluate

$$\int_0^\infty \frac{d\omega'}{(\omega'^2 - \omega^2)^2} \operatorname{Im}\left(-\frac{1}{\epsilon_L(\omega')}\right) \quad \text{— sing. contrib.}$$

Lots of sum rules, eg.

$$\frac{2}{\pi} \int \omega' \, d\omega' \operatorname{Im}\left(-1/\epsilon(\omega')\right) = \omega_p^2$$

$$\frac{2}{\pi} \int \frac{d\omega'}{\omega'} \operatorname{Im}\left(-1/\epsilon(\omega')\right) = 1$$

so, generically, expect $\int \sim \omega_p^{-2}$ and hence $\Psi(\omega) \sim 1$.

However, a surprise:

for simple Drude model ($\sigma(\omega) = \epsilon_0 \omega_p^2/(1 - i\omega\tau)$) can evaluate $\Psi(\omega)$ for arbitrary ω. Then if we take the limits

$$\tau^{-1} \ll \omega \ll \omega_p$$

(but no restriction on ratio of small quantities $\omega/\omega_p, 1/\omega\tau$) we find

$$\Psi(\omega) = 1 - \frac{\omega_p}{2\pi\omega^2\tau} + O(1/\omega_p\tau)$$

For currently realistic expt. params, $\omega_p/2\pi\omega^2\tau < 1$. But need not be so $\Rightarrow$ $J_{mw}(t)$ can be -ve!

OK, bec. only physically meaningful quantity is $J(t) \equiv J_{mw}(t) + J_{imw}(t)$, and this is always +ve.

In any case, we expect

$$\Psi(\omega) \lesssim 1,$$
$$\Phi(\omega) \lesssim 1$$

and so, just as in simple "London" model,

$$J \lesssim (\epsilon_0 c/\pi\hbar)|A_0|^2$$

Now the crunch: determine A_0 from the matching conditions at 1-2 interface and the consideration that total (extra) field en. = $\hbar\omega$.

The net result: for a S.P.,

$$|A_0|^2 \cong \frac{\hbar}{\epsilon_0 c}\,\frac{\epsilon_1}{|\epsilon_2|}\cdot\frac{1}{(|\epsilon_1+\epsilon_2|)^{1/2}}$$

so

$$J \cong \frac{1}{\pi}\cdot\frac{\epsilon_1}{|\epsilon_2|}\cdot\frac{1}{(|\epsilon_1+\epsilon_2|)^{1/2}}$$

But for $\omega \ll \omega_p$ (explain. radiation cond^n?)

$$\epsilon_1 \sim 1,\quad \epsilon_2 \sim -\frac{\omega_p^2}{\omega^2}.$$

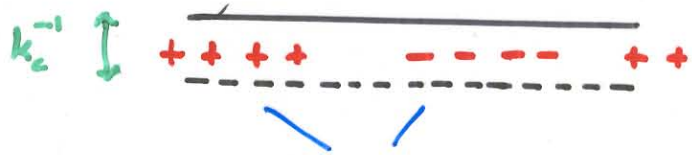
so

$$\boxed{J \sim (\omega/\omega_p)^3}$$

In expt. of Farad et al., metal is Au ($\omega_p \cong 8$ eV) and ω is in Telecom band (1500 nm $\cong$ 0·75 eV), so

$$J \sim 10^{-3}.$$

How can this be?



~ 300 e^-'s in total!

Regard surface layer as II-plate capacitor, calculate **zero-point** fluct^n in absence of S.P.:

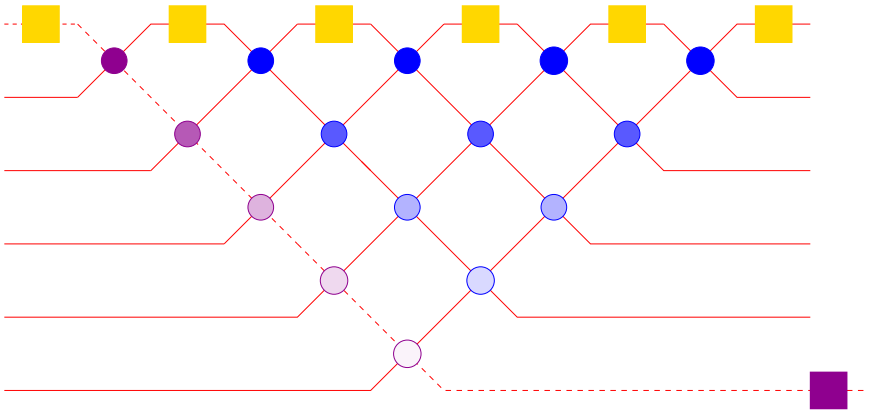
$$Q^2/2C = \tfrac{1}{4}\hbar\omega_p \Rightarrow N \sim \left(\frac{\epsilon_0}{e^2}k_c L^2\cdot\hbar\omega_p\right)^{1/2} \sim L/a_0$$

(Bohr radius)

$\Rightarrow$

$$N_{zp} \sim 10^7\ !$$

Things I wish they had told me about Shor's factoring algorithm



Vinay Ambegaokar's Retirement Symposium

Cornell, June 16, 2007

Question:

What has quantum mechanics to do with factoring?

Answer:

Nothing!

But quantum mechanics is good at diagnosing periodicity, which (for purely arithmetic reasons) helps in factoring.

Factoring $N = pq$, where p and q are huge (e.g. 300 digit) primes, follows from ability to find smallest r with $a^r = 1 \pmod N$ for integers a sharing no factors with N.

$a^x \pmod N$ is periodic with period r.

Pick random a. Use quantum computer to find r.

Pray for two pieces of good luck.

Quantum computer gives least r with $a^r - 1$ divisible by $N = pq$

First piece of luck: r even.
Then $(a^{r/2} - 1)(a^{r/2} + 1)$ is divisible by N, but $a^{r/2} - 1$ is *not*,

Second piece of luck: $a^{r/2} + 1$ is *also* not divisible by N.
Then product of $a^{r/2} - 1$ and $a^{r/2} + 1$ is divisible by both p and q although neither factor is divisible by both.

Since p, q primes, one factor divisible by p and other divisible by q.
So p is greatest common divisor of N and $a^{r/2} - 1$
and q is greatest common divisor of N and $a^{r/2} + 1$

FINISHED!

Finished, *because:*

1. Can find greatest common divisor of two integers using simple method known to ancient Greeks: Euclidean algorithm.

2. If a is picked at random, an hour's argument shows that the probability is at least 50% that both pieces of luck will hold.

Amazing (but wrong):

[After the computation] the solutions — the factors of the number being analyzed — will all be in superposition.

— George Johnson, *A Shortcut Through Time.*

[The computer will] try out all the possible factors simultaneously, in superposition, then collapse to reveal the answer.

— *Ibid.*

Unexciting but correct:

A quantum computer is efficient at factoring because it is efficient at period-finding.

Next question: What's so hard about period finding?

Given graph of $\sin(kx)$ it's easy to find the period $2\pi/k$. Since no value repeats inside a period, $a^x \pmod N$ is even simpler.

What makes it hard:

Within a period, unlike the smooth, continuous $\sin(kx)$, the function $a^x \pmod N$ looks like random noise.

Nothing in a list of r consecutive values gives a *hint* that the next one will be the same as the first.

PERIOD FINDING WITH A QUANTUM COMPUTER

Represent n bit number

$$x = x_0 + 2x_1 + 4x_2 + \cdots + 2^{n-1}x_{n-1} \qquad (\text{each } x_j \text{ 0 or 1})$$

by product of states $|0\rangle$ and $|1\rangle$ of n 2-state systems (*Qbits*):

$$|x\rangle = |x_{n-1}\rangle \cdots |x_1\rangle |x_0\rangle$$

Classical or *Computational* basis.

Qbits, not *qubits* because:

1. Classical two state systems are *Cbits* (not *clbits*)
2. Ear cleaners are *Qtips* (not *Qutips*)
3. Dirac wrote about *q-numbers* (not *qunumbers*)

(*q-bit* awkward: *2-Qbit gate* OK;
2-q-bit gate unreadable.)

Represent function f taking n-bit to m-bit integers
by a linear, norm-preserving (unitary) transformation $\mathbf{U}_f$
acting on n-Qbit *input register* and m-Qbit *output register*:

input register

$$\mathbf{U}_f|x\rangle|0\rangle = |x\rangle|f(x)\rangle.$$

output register

$$\mathbf{U}_f|x\rangle|0\rangle = |x\rangle|f(x)\rangle$$

Put input register into superposition of all possible inputs:

$$|\phi\rangle = \left(\tfrac{1}{\sqrt{2}}\right)^n \sum_{0\le x<2^n} |x\rangle$$

$$= \tfrac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big)\cdots\tfrac{1}{\sqrt{2}}\big(|0\rangle + |1\rangle\big).$$

Applying linear $\mathbf{U}_f$ gives

$$\mathbf{U}_f\big(|\phi\rangle|0\rangle\big) = \left(\tfrac{1}{\sqrt{2}}\right)^n \sum_{0\le x<2^n} |x\rangle|f(x)\rangle.$$

$$\mathbf{U}_f\big(|\phi\rangle|0\rangle\big) = \big(\tfrac{1}{\sqrt{2}}\big)^n \sum_{0\le x<2^n} |x\rangle|f(x)\rangle.$$

Special form when $f(x) = a^x \pmod N$:

$$\sum_{0\le x<2^n} |x\rangle|a^x\rangle = \sum_{0\le x<r} \Big(|x\rangle + |x+r\rangle + |x+2r\rangle + \cdots\Big)|a^x\rangle$$

Measuring output register leaves input register in state

$$|x\rangle + |x+r\rangle + |x+2r\rangle + \cdots$$

for random $x < r$.

THE QUANTUM FOURIER TRANSFORM (QFT)

$$\mathbf{V}_{FT}|x\rangle = \left(\tfrac{1}{\sqrt{2}}\right)^n \sum_{0\le y<2^n} e^{2\pi i x y/2^n}|y\rangle$$

Acting on superpositions, $\mathbf{V}_{FT}$ Fourier-transforms amplitudes:

$$\mathbf{V}_{FT}\sum \alpha(x)|x\rangle = \sum \beta(x)|x\rangle$$

$$\beta(x) = \left(\tfrac{1}{\sqrt{2}}\right)^n \sum_{0\le z<2^n} e^{2\pi i x z/2^n}\alpha(z)$$

If α has period r, as in $|x\rangle + |x+r\rangle + |x+2r\rangle + \cdots$, then β is sharply peaked at integral multiples of $2^n/r$.

HO-HUM!

$\mathbf{V}_{FT}$ is *boring*:

1. Just familiar transformation from
   position to momentum representation.
2. Everybody knows Fourier transform
   sharply peaked at multiples of inverse period.

But $\mathbf{V}_{FT}$ is *not* ho-humish because:

1. x has nothing to do with position, real or conceptual.

x is arithmetically useful but physically meaningless:

$$x = x_0 + 2x_1 + 4x_2 + 8x_3 + \cdots,$$

where $|x_j\rangle = |0\rangle$ or $|1\rangle$ is state of j-th 2-state system.

2. *Sharp* means sharp compared with resolution of apparatus.

The period r is hundreds of digits long.

Need to know r *exactly* — every single digit.

Error in r of 1 in 10^{10} messes up almost every digit.

$$\mathbf{V}_{FT}\Big(|x\rangle + |x+r\rangle + |x+2r\rangle + \cdots\Big) =$$

$$= \big(\tfrac{1}{\sqrt{2}}\big)^n \sum_{0\le y<2^n} \Big(1 + \alpha + \alpha^2 + \alpha^3 + \cdots\Big) e^{2\pi i x y/2^n} |y\rangle,$$

$$\alpha = \exp\Big(2\pi i y/(2^n/r)\Big).$$

Sum of phases α sharply peaked at values of y as close as possible to (i.e. within $\frac{1}{2}$ of) integral multiples of $2^n/r$.

Question: *How sharply peaked?*

Answer: Probability that measurement of input register gives such a value of y exceeds 40%.

Good ($> 40\%$) chance of getting integer y within $\frac{1}{2}$ of $j(2^n/r)$ for some (more or less) random integer j.

Then $y/2^n$ is within $1/2^{n+1}$ of j/r.

Question: Does this pin down unique rational number j/r?

Answer: Yes, if $2^n > r^2$.

Since $r < N$, input register must be large enough to represent N^2. Must contain at least N periods.

Then have 40% chance of learning a *divisor* r_0 of r.

(r_0 is r divided by factors it shares with (random) j)

Comment:

Should the period r be 2^m, then $2^n/r$ is itself an integer, and probability of y being multiple of that integer is easily shown to be 1, even if input register contains just a single period.

A pathologically easy case.

Question: When must all periods r be powers of 2?
Answer: When p and q are both of form $2^j + 1$.
(Periods are divisors of $(p-1)(q-1)$.)

Therefore factoring $15 = (2+1) \times (4+1)$
— i.e. finding periods modulo 15 —
is not a serious demonstration of Shor's algorithm.

Some neat things about the quantum Fourier transform

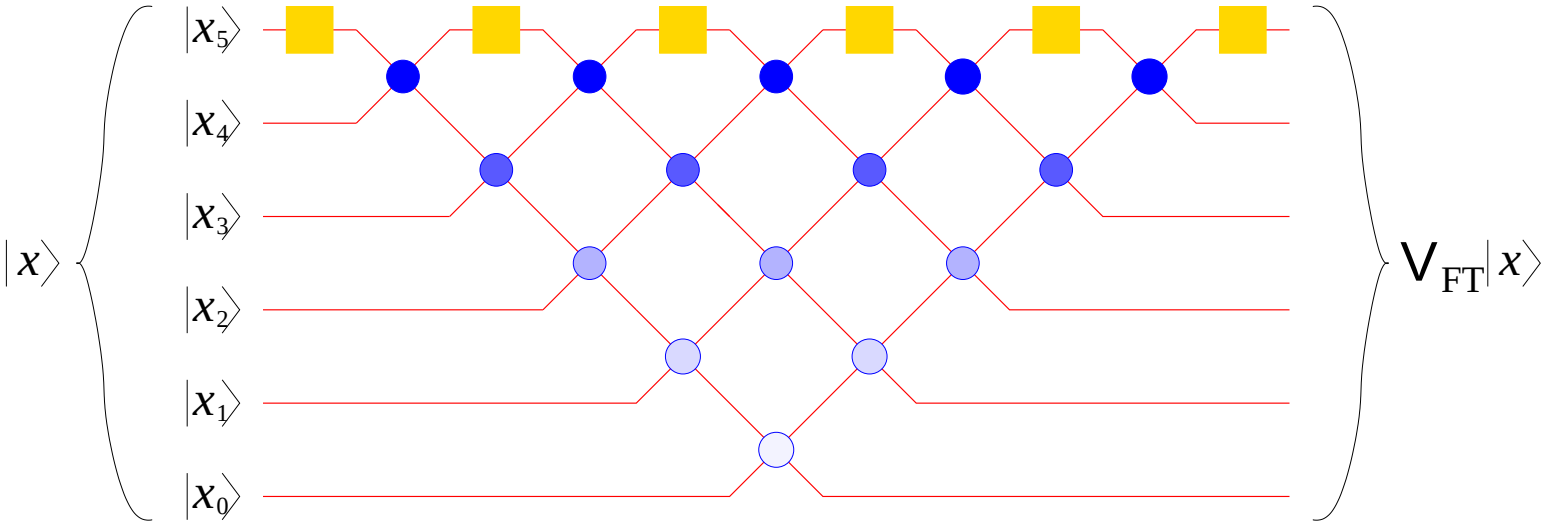
$$\mathbf{V}_{FT}|x\rangle = \left(\tfrac{1}{\sqrt{2}}\right)^n \sum_{0\le y<2^n} e^{2\pi i x y/2^n}|y\rangle$$

1. Constructed entirely out of 1-Qbit and 2-Qbit gates.
2. Number of gates and therefore time grows only as n^2.
3. With just *one* application,

$$\sum \alpha(x)|x\rangle \longrightarrow \sum \beta(x)|x\rangle,$$

$$\beta(x) = \left(\tfrac{1}{\sqrt{2}}\right)^n \sum_{0\le z<2^n} e^{2\pi i x z/2^n}\alpha(z)$$

In *classical* "Fast Fourier Transform" time grows as $n2^n$.
But classical FFT gives all the $\beta(x)$, while QFT only gives $\sum \beta(x)|x\rangle$.

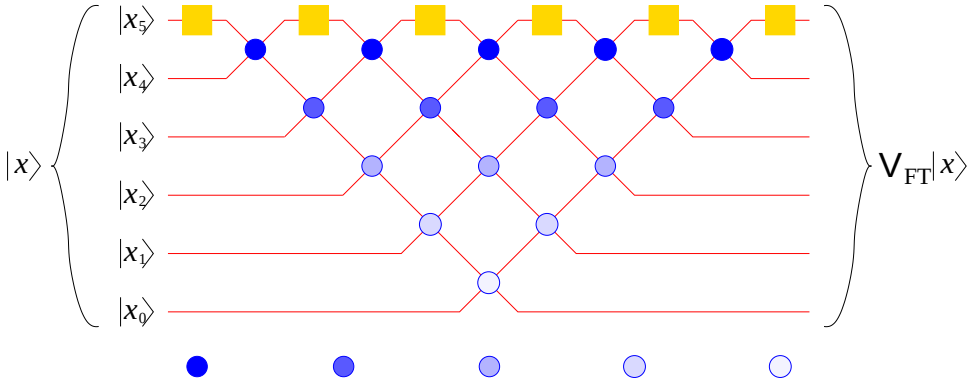


$$\left.\begin{matrix}|0\rangle \\ |1\rangle\end{matrix}\right\} \quad \left\{\begin{matrix}\frac{1}{\sqrt{2}}(|0\rangle + |1\rangle) \\ \frac{1}{\sqrt{2}}(|0\rangle - |1\rangle)\end{matrix}\right.$$

$e^{\pi i \mathbf{nn}'/2}$ $e^{\pi i \mathbf{nn}'/4}$ $e^{\pi i \mathbf{nn}'/8}$ $e^{\pi i \mathbf{nn}'/16}$ $e^{\pi i \mathbf{nn}'/32}$

$|0\rangle|0\rangle,\ |0\rangle|1\rangle,\ |1\rangle|0\rangle$ invariant; $\quad |1\rangle|1\rangle \longrightarrow e^{\pi i/2^j}|1\rangle|1\rangle$

A PROBLEM?



Number n of Qbits: $2^n > N^2$, N hundreds of digits.

Phase gates $e^{\pi i \mathbf{nn}'/2^m}$ *impossible to make* for most m,

since can't control strength or time of interactions to better than parts in $10^{10} = 2^{30}$.

But need to learn period r to parts in 10^{300} or more!

Question:

So is it all based on a silly mistake?

Answer:

No, all is well.

Question:

How can that be?

Answer:

Because of the quantum-computational interplay between analog and digital.

Quantum Computation is Digital

Information is acquired *only* by measuring Qbits.
The reading of each 1-Qbit measurement gate
is only 0 or 1.

The 10^3 bits of the output y of Shor's algorithm
are given by the readings (0 or 1) of 10^3
1-Qbit measurement gates.

There is no imprecision in those 10^3 readings.
The output is a definite 300-digit number.

But is it the number you wanted to learn?

Quantum Computation is Analog

Before a measurement the Qbits
are acted on by unitary gates with
continuously variable parameters.

These variations affect the amplitudes
of the states prior to measurement
and therefore they affect the *probabilities*
of the readings of the measurement gates.

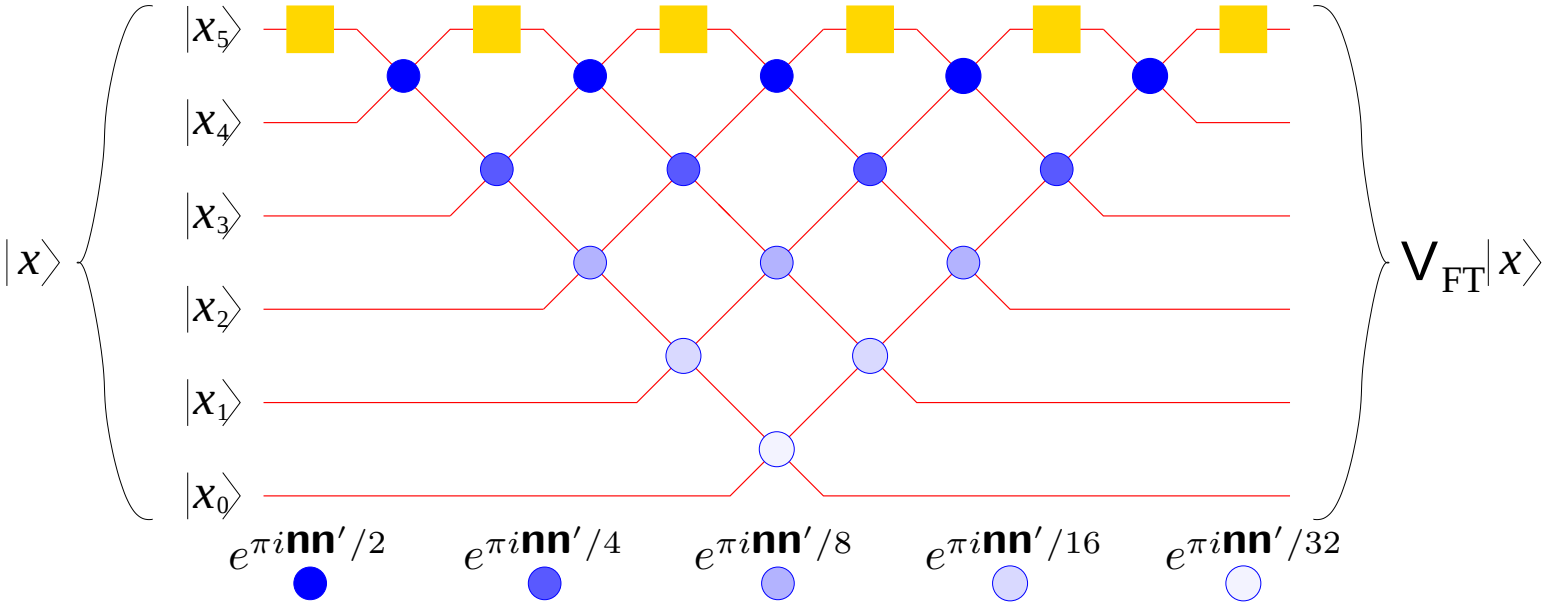
So all is well

"Huge" errors (parts in 10^4) in the phase gates
may result in comparable errors in the *probability* that
the 300 digit number given *precisely* by the measurement gates
is *the right* 300 digit number.

So the probability of getting a useful number
may not be 40% but only 39.99%.

Since "40%" is actually "about 40%"
this makes no difference.

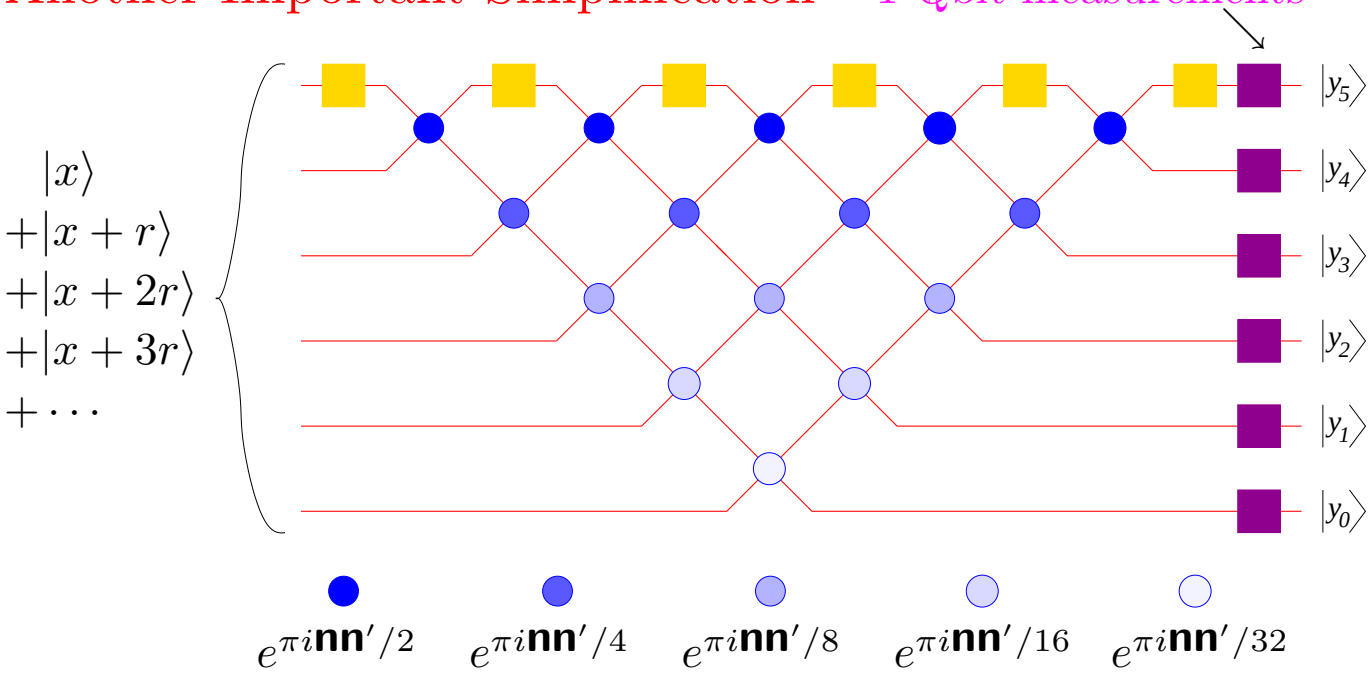
In fact this makes things even better



Since only top 20 layers of phase gates matter, when $N > 2^{20} = 10^6$, *running time scales not quadratically but linearly in number of Qbits.*

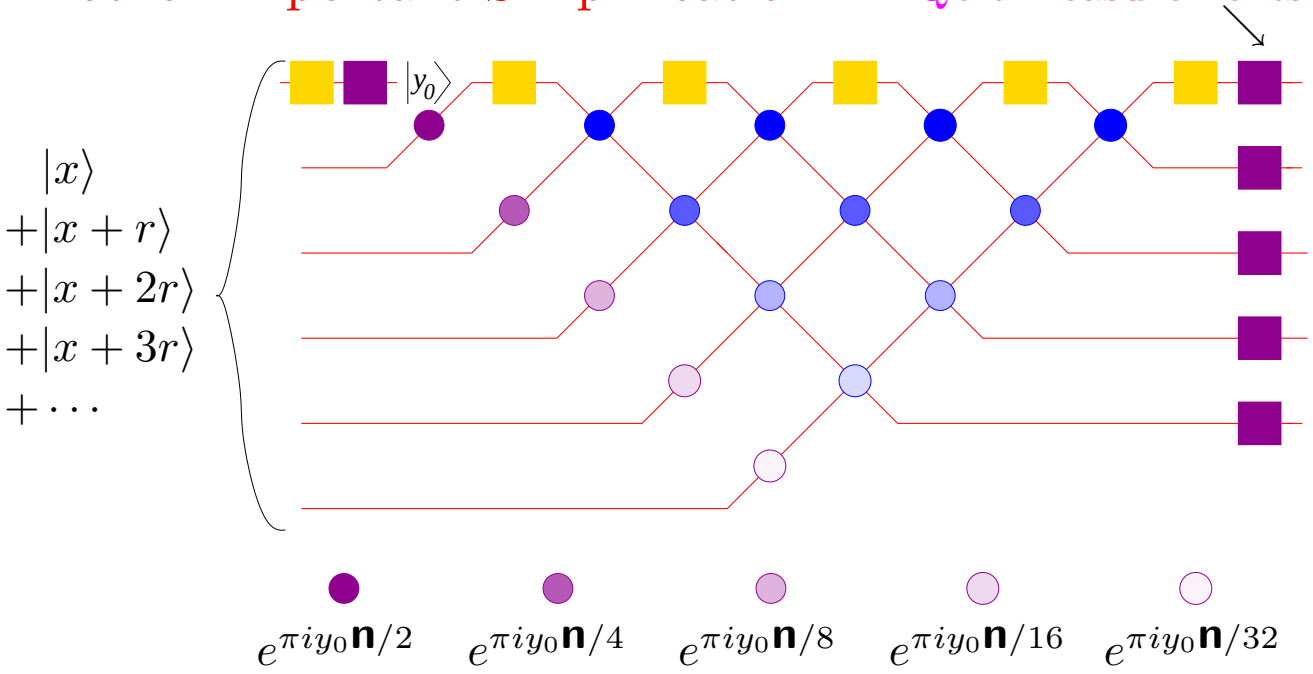
Another Important Simplification

1-Qbit measurements



Another Important Simplification

1-Qbit measurements



$e^{\pi i y_0 \mathbf{n}/2}$ $e^{\pi i y_0 \mathbf{n}/4}$ $e^{\pi i y_0 \mathbf{n}/8}$ $e^{\pi i y_0 \mathbf{n}/16}$ $e^{\pi i y_0 \mathbf{n}/32}$

You don't need anything but 1-Qbit gates!

References:

Cornell lecture notes
[Google: `mermin CS483`*]*

Quantum Computer Science
N. David Mermin
Cambridge University Press
August 2007 (I hope)

Quantum Versus Classical Programming Styles

Question: How do you calculate a^x *when* x *is a 300 digit number?*
Answer: Not by multiplying a *by itself* 10^{300} *times!*

How else, then?

Write x *as a binary number:* $x = x_{999}x_{998}\cdots x_2x_1x_0$.

Next square a*, square the result, square that result . . . , getting the 1,000 numbers* a^{2^j} .

Finally, multiply together all the a^{2^j} *for which* $x_j = 1$.

$$\prod_{j=0}^{999}\left(a^{2^j}\right)^{x_j} = a^{\sum_j x_j 2^j} = a^x$$

Classical: Cbits Cheap; Time Precious

$$a^x = \prod_{j=0}^{999} \left(a^{2^j}\right)^{x_j}$$

Once and for all, make and store a look-up table:

$$a,\ a^2,\ a^4,\ a^8, \ldots,\ a^{2^{999}}$$

A thousand entries, each of a thousand bits.

For each x multiply together all the a^{2^j} in the table for which $x_j = 1$.

Quantum: Time Cheap; Qbits Precious

Circuit that executes

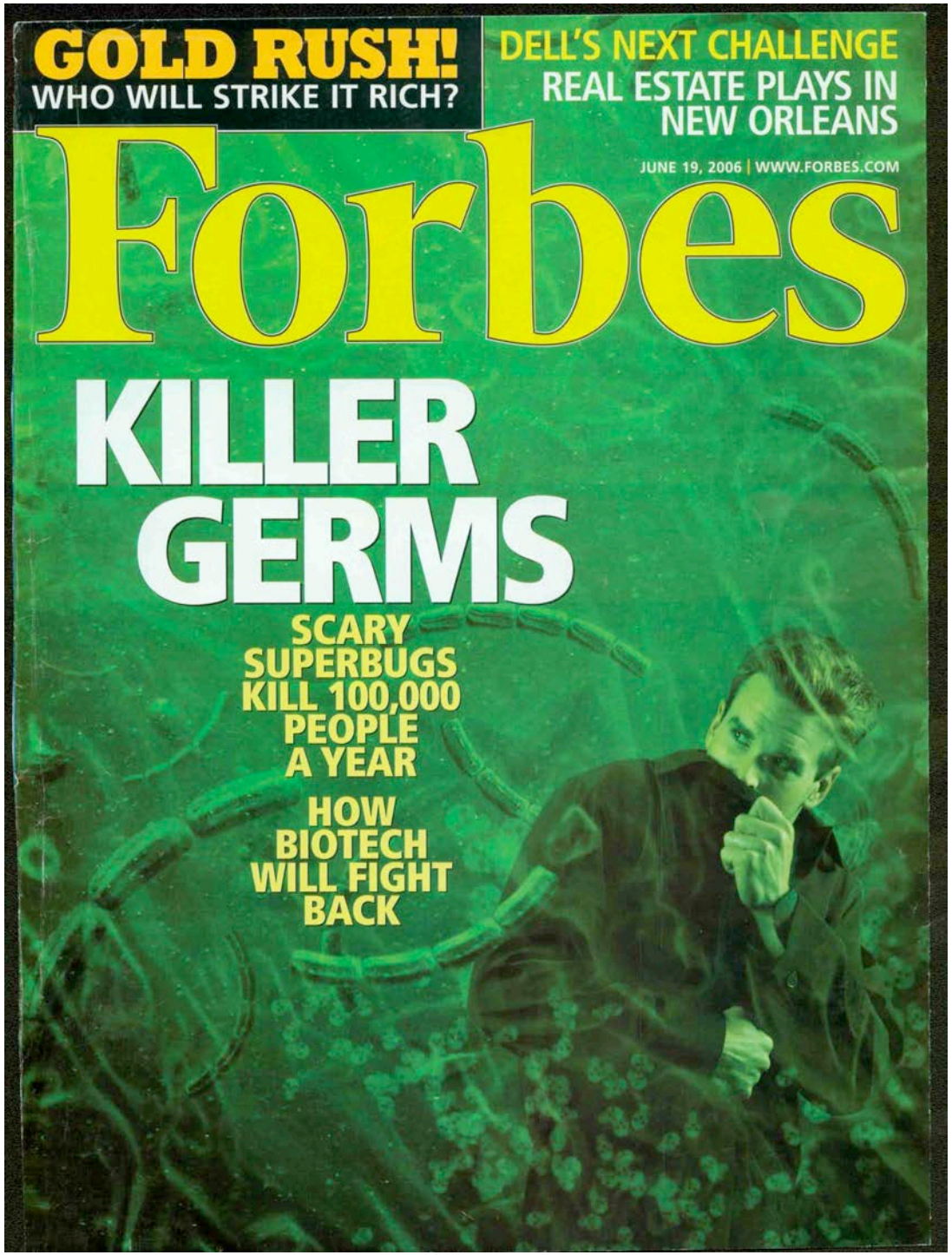
$$a^x = \prod_{j=0}^{999} \left(a^{2^j}\right)^{x_j}$$

is not applied 2^n *times to input register for each* $|x\rangle$.
It is applied just once to input register in the state

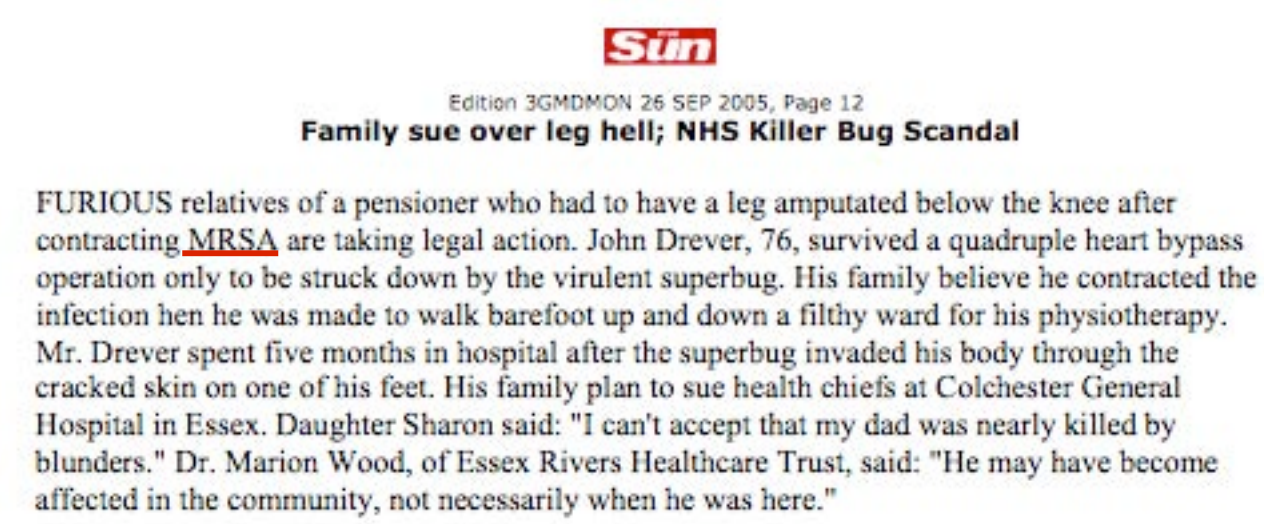
$$|\phi\rangle = \left(\tfrac{1}{\sqrt{2}}\right)^n \sum_{0 \le x < 2^n} |x\rangle.$$

So after each conditional (on $x_j = 1$*) multiplication by* a^{2^j}
can store $\left(a^{2^j}\right)^2 = a^{2^{j+1}}$ *using same 1000 Qbits*
that formerly held a^j.

Evolution of antibiotic resistance by whole genome resequencing



Staphylococcus aureus and British elections I



Sun

Edition 3GMDMON 26 SEP 2005, Page 12

Family sue over leg hell; NHS Killer Bug Scandal

FURIOUS relatives of a pensioner who had to have a leg amputated below the knee after contracting MRSA are taking legal action. John Drever, 76, survived a quadruple heart bypass operation only to be struck down by the virulent superbug. His family believe he contracted the infection hen he was made to walk barefoot up and down a filthy ward for his physiotherapy. Mr. Drever spent five months in hospital after the superbug invaded his body through the cracked skin on one of his feet. His family plan to sue health chiefs at Colchester General Hospital in Essex. Daughter Sharon said: "I can't accept that my dad was nearly killed by blunders." Dr. Marion Wood, of Essex Rivers Healthcare Trust, said: "He may have become affected in the community, not necessarily when he was here."

Methacillin Resistant S.aureus (MRSA) enters the popular press

S.aureus and British elections II

Sun

Edition 3GMDMON 26 SEP 2005, Page 12

I caught MRSA 9 times in the same hospital; NHS Killer Bug Scandal

EMMA MORTON, HEALTH REPORTER

AN EX-trucker told last night how he lost his right knee after catching the MRSA superbug NINE TIMES in hospital.

Ken Sutton, 60, was repeatedly hit by the deadly infection after an op to rebuild his leg following a motorbike smash.

The bug -which he blames on hygiene blunders -crippled him for life, wrecked his marriage and left him unable to work again.

Ken said last night: "My life is ruined. I had a good job, a loving wife and house of my own and I have lost everything."

"Now I live in a disabled bungalow on Pounds 80 benefits a week and cannot walk properly."

He added: "Having MRSA once is a nightmare -imagine nine times over two years."

"I'd wake up at night screaming in pain. Passers-by in the street would hear and bang on the door."

"I've had a raw deal. To catch it over and over is an appalling indictment of the NHS."

S.aureus and British elections III

Edition 2GMDMON 22 MAR 2004, Page 1

NHS killer bug shock;Sun investigation;Exclusive

ANTHONY FRANCE

We find 80 times danger level of MRSA in hospital

EIGHTY times the danger level of killer superbug MRSA were found at an NHS hospital in a shock investigation by The Sun.

A reporter took samples while working undercover at the North Middlesex Hospital in Edmonton, North London -named a year ago as the joint worst infected in Britain. Bosses vowed then to improve hygiene. But microbiologist Christopher Malyszewicz said of our investigation: "These are the worst results I've seen. It is frightening. The hospital is failing to tackle the problem."

Our reporter took 24 swabs from toilets, soap dispensers, lifts and banisters. An average 40 colony forming units (CFU) were found growing per square centimetre. A reading of 0.53 CFUs is regarded as "the base line" -and is still a risk to patients and visitors.

Mr Malyszewicz, whose company Chemsol tested our samples, said: "In some areas we found 80 times the level which has normally been found, which is unusually high and very dangerous.

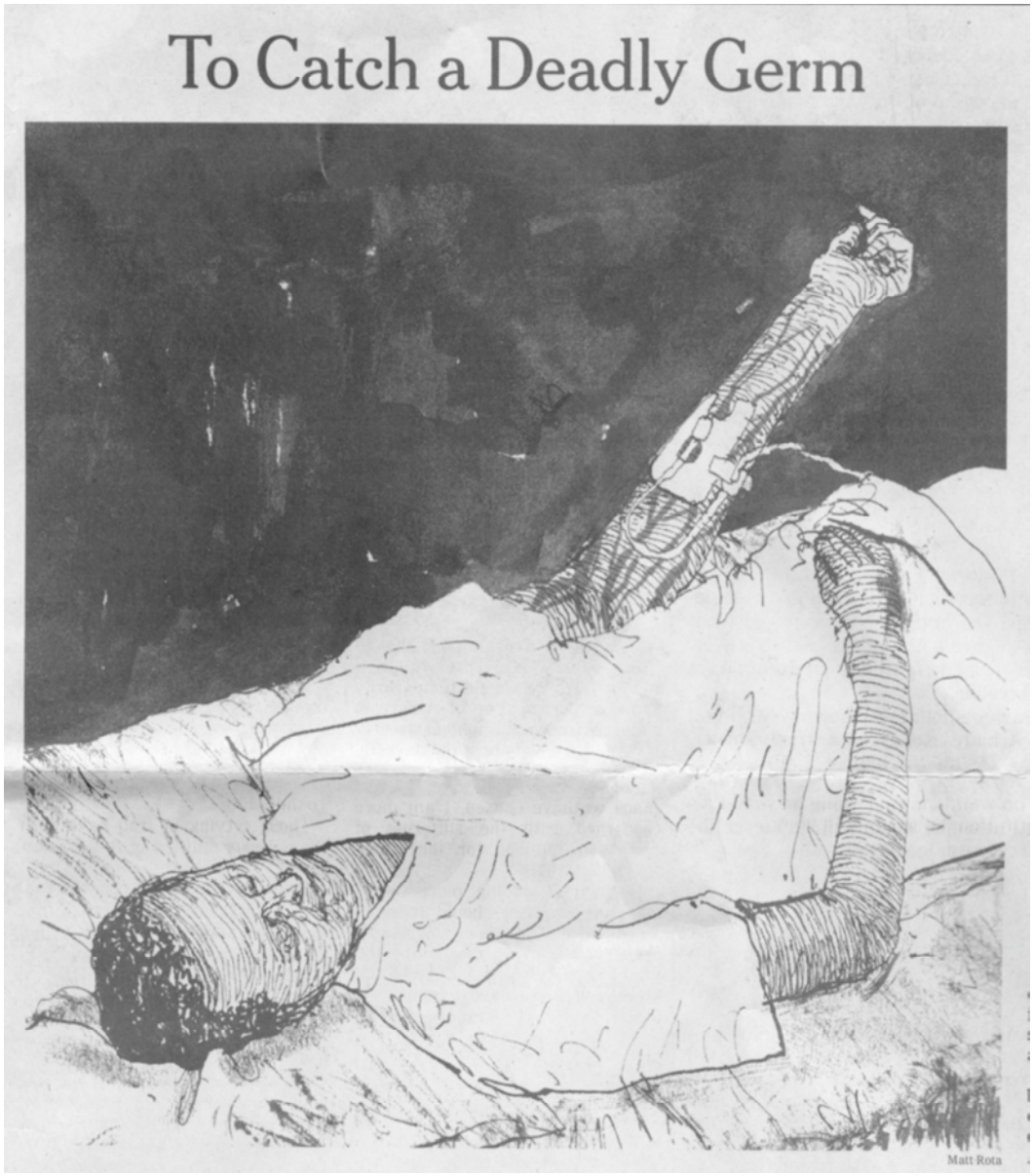
Deadly

"Almost every swab you had taken had MRSA present, which is unusual."

"This means MRSA is widespread throughout the hospital and not just in isolated patches."

Deaths from MRSA -Methicillin-resistant Staphylococcus Aureus –have risen 15-fold nationally in the past decade. In 2002, 800 people died, compared with 51 in 1993.

NYTimes 11/14/06 OP-ED



- MRSA kills 5x more americans than HIV, 60% S.aureus infections resistant, cost of hospital infections $3 10^{10}.
- Spread via bedrails, wheelchairs, door knobs, stethoscopes, lab coats, blood pressure cuffs (77% contaminated in Fr Hospital)..
- Screen, isolate patients can reduce infections 90%. Select your hospital carefully or go to Denmark.

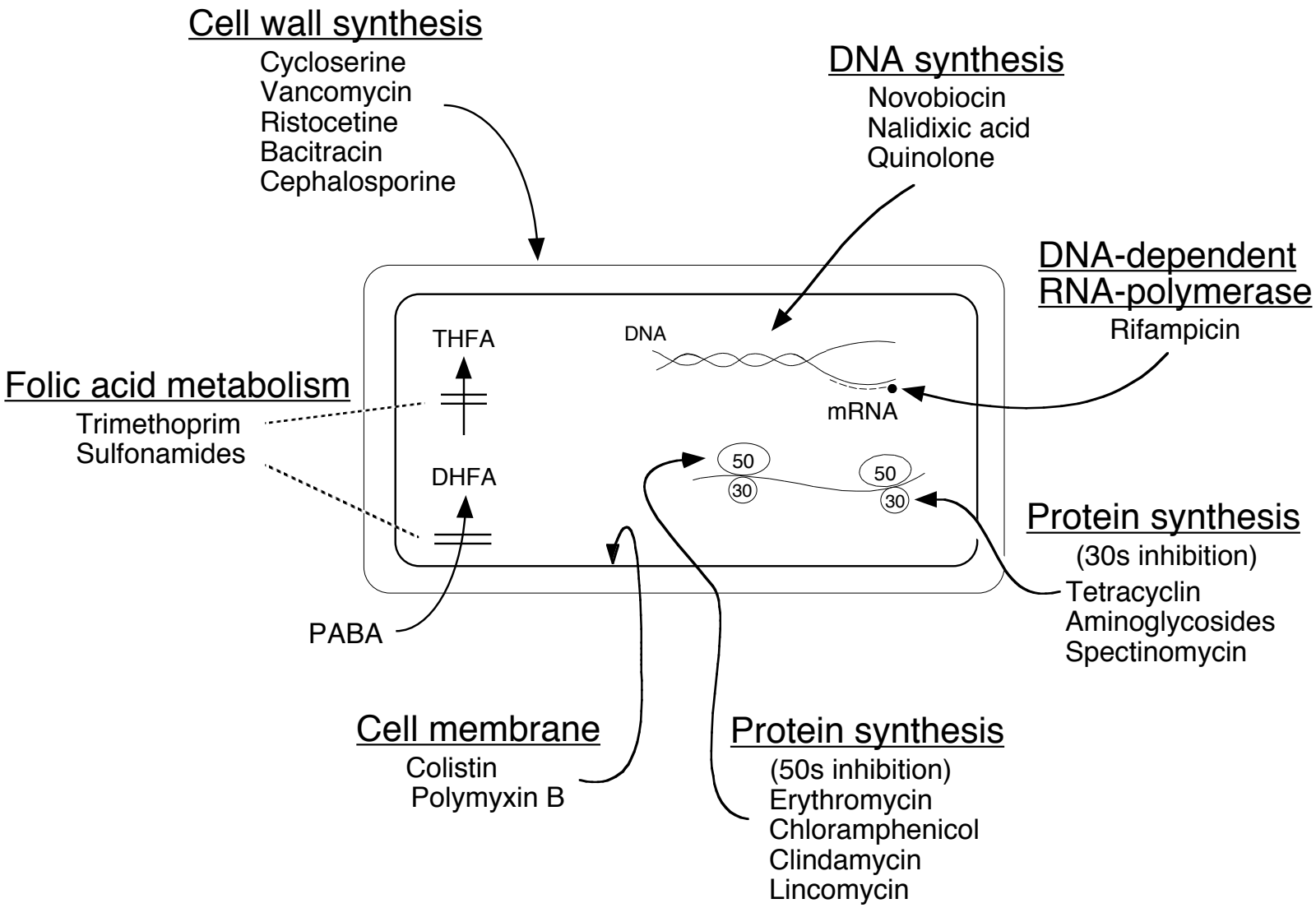
Antibiotics

Most antibiotics are natural products, or derivatives, hence while we prospect the biosphere for novel compounds, the bacteria do likewise for novel defense/signaling mechanisms.*

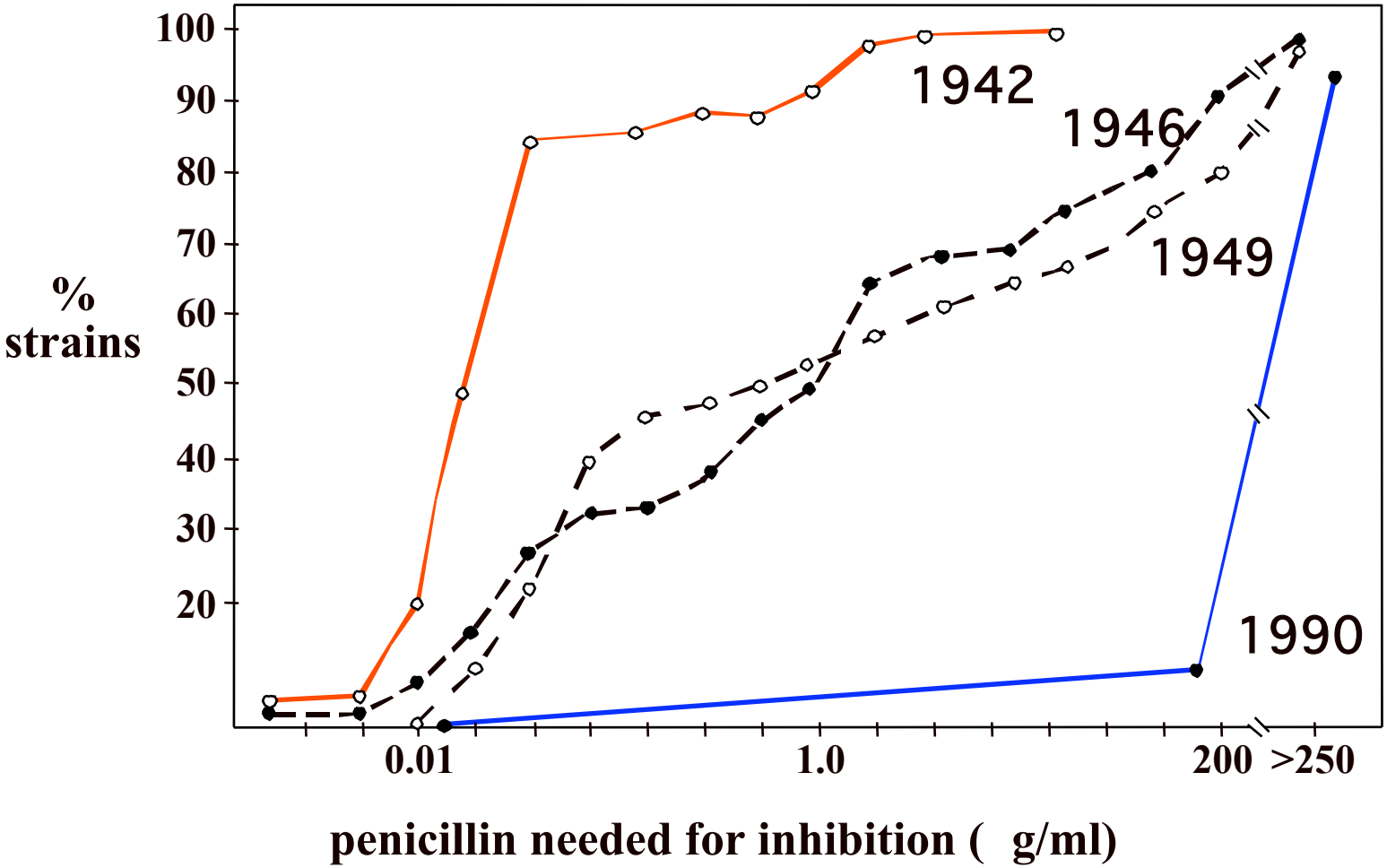
Strategies:
- pump it out or restrict intake,
- chemically inactivate it,
- modify the target gene by point mutations,
- use completely new genes for targeted function

* A Tomasz, *Science* **311** 342 2006

Antibiotics target many pathways

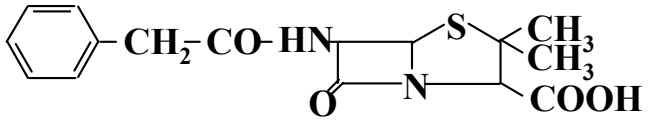


Penicillin resistance

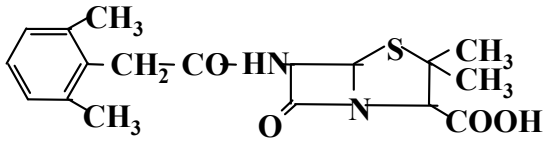


Penicillin derivatives, beta-lactams, also induced resistance

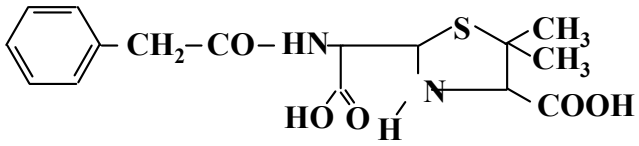
1943: Penicillin in therapy



1960: Methicillin in therapy (resistant to Penicillinase)



1946-1950:import Penicillinase gene



1961: import *mecA* gene, does not bind b-lactams

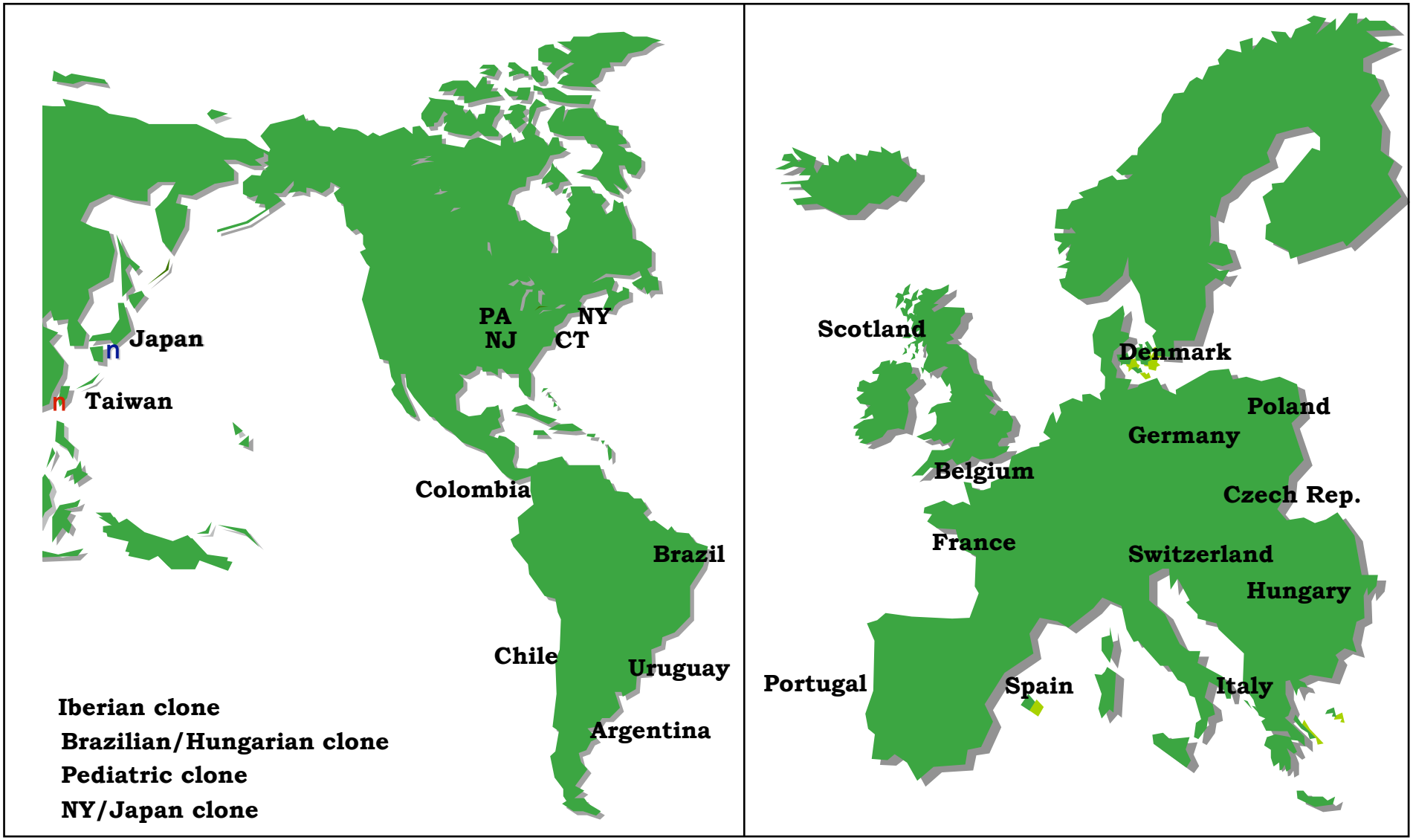
Methicillin resistant strains become global by 1980s

1960: Penicillin resistant staphylococci become global through plasmid epidemics

Spread of antibiotic resistance

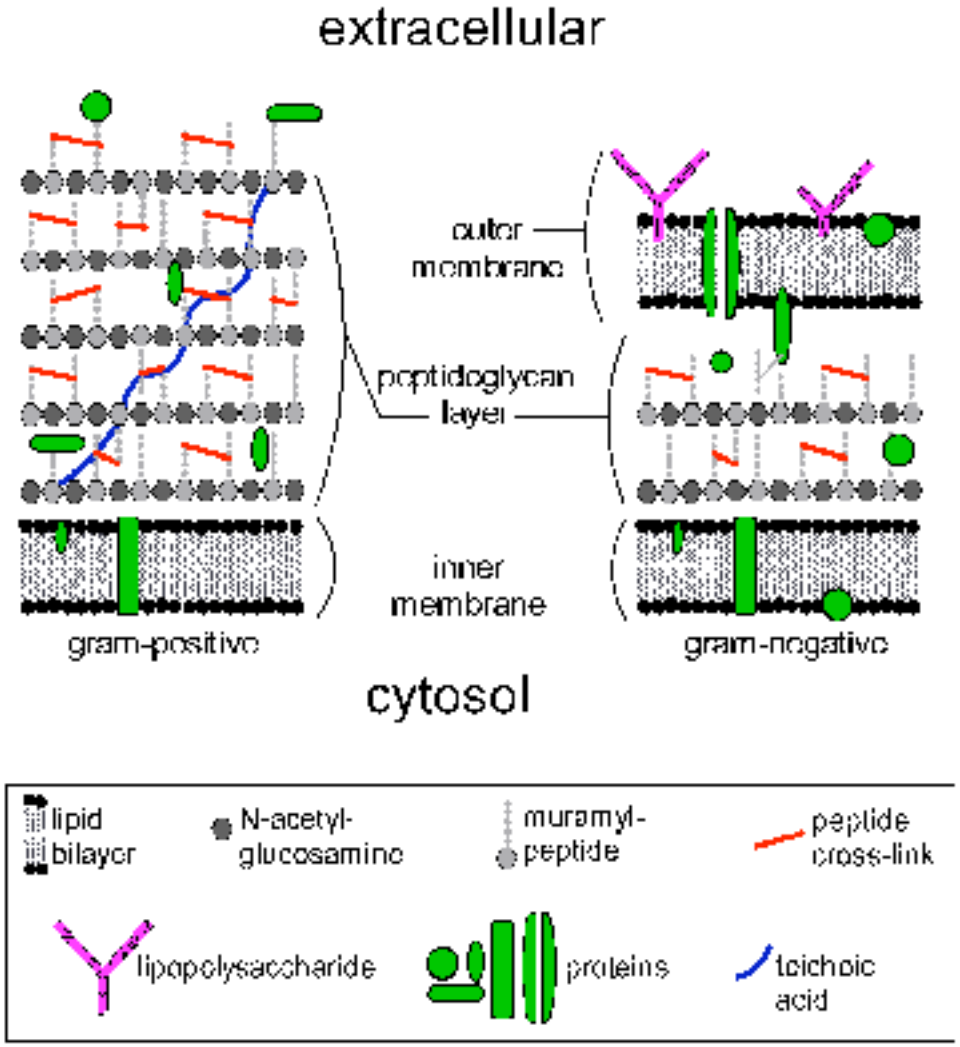
	S. aureus ATCC 6538 (1930)	MRSA Brazilian epidemic Clone (1994)	Resistance mechanism acquired (+) adaptive (A)
Penicillin	S	R	+(1945)
Streptomycin	S	R	+ (1948)
Tetracycline	S	R	+ (1950)
Methicillin	S	R	+ (1961) *mecA*
Oxacillin	S	R	
Cephalothin	S	R	
Cefotaxime	S	R	
Imipenem	S	R	
Chloramphenicol	S	R	+
Ciprofloxacin	S	R	A
Clindamycin	S	R	+
Erythromycin	S	R	+
Gentamycin	S	R	+
Rifampin	S	R	A
Vancomycin	S	S	A (1997) VISA
Vancomycin	S	S	+ (2002) *vanA*
Teicoplanin	S	S	+
Trimeth/Sulfa	S	R	A
Mupirocine (topical)	S	R	+

Geographic Spread of Pandemic MRSA Clones



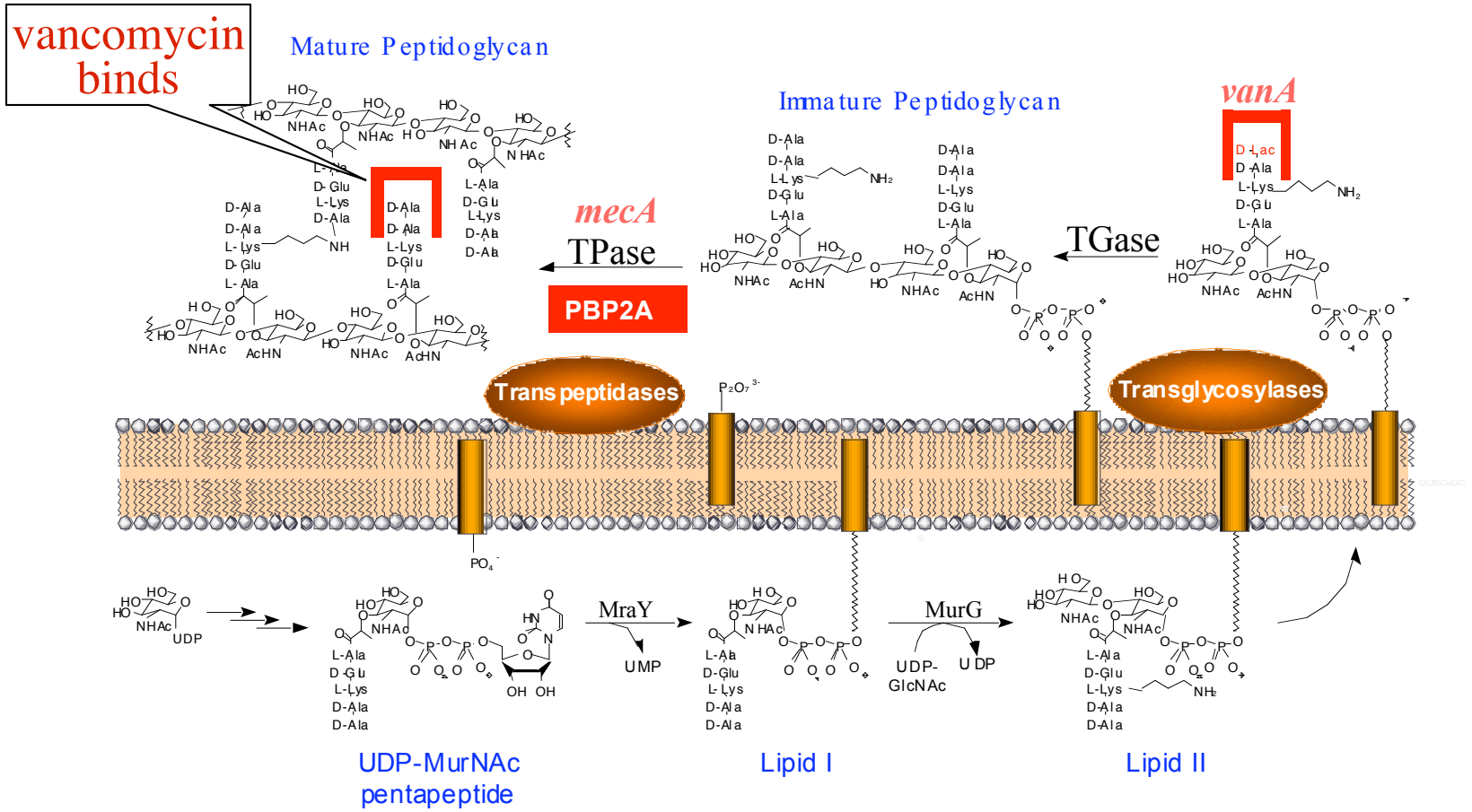
Oliveira et al. Lancet 2002. **2**:180-189

Primer on bacterial cell walls



Cell Wall Biosynthesis

Peptidoglycan Biosynthesis



Cell wall specific antibiotics

beta-lactams (eg methacillin) inhibit the enzymes that link the protein and sugars in the cell wall. Resistance requires foreign gene mecA.

Vancomycin: Backup treatment, binds to cell wall precursors

- Resistance: Adaptive evolution by several mutations, general 'stress response' thickened cell wall, problems dividing, slow growth. Genes involved?
- Population response: hetero-resistance strategy, how to prosper in environments +- vanco when resistance imposes growth penalty: spontaneously mutate $1{:}10^5$ bugs, fitness cost $1\text{-}10^{-5}$. Evolution by design?

Altered cell wall & morphology in VISA strain

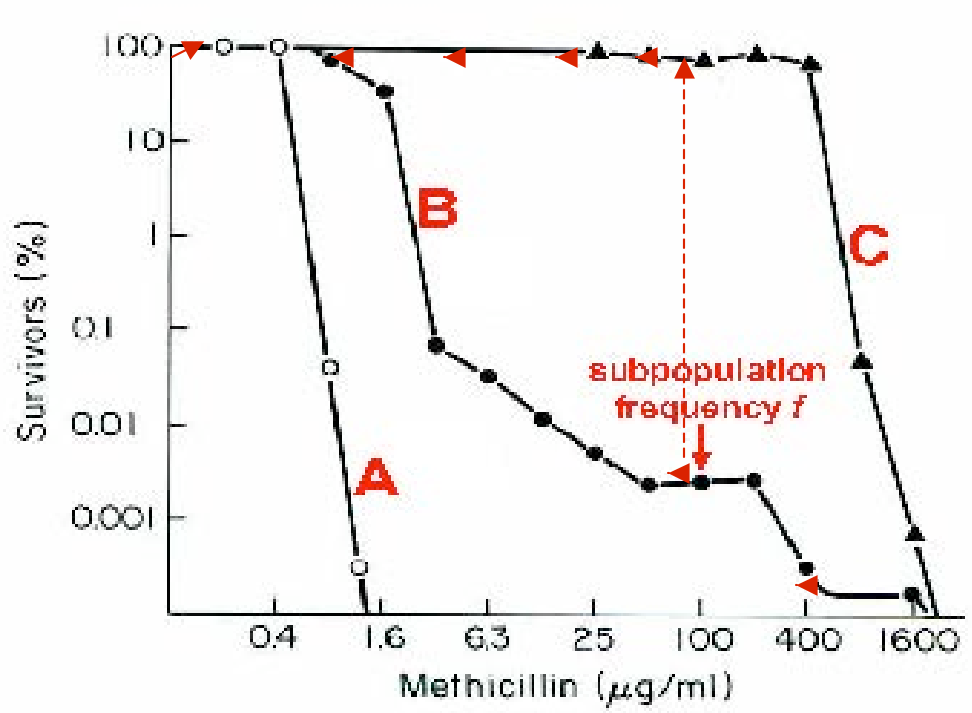
wt VISA



wt VISA

Heteroresistance

Amplify bugs good media (no ab), test fraction that grow on plates with incr ab



A: susceptible, B: heteroresistant, C homogeneous resistance

Pick colony from B, re-grow permissive media -> homogeneous resistant to original level and trait genetic.

Resistance -> growth penalty, hetero-resistance good evolutionary strategy.

mecA gene is associated with large heterologous DNA cassettes

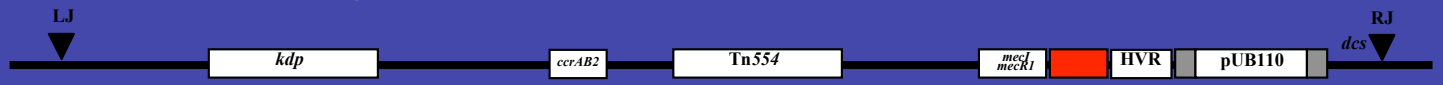
SCC*mec* type I



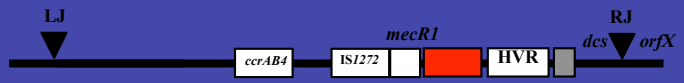
SCC*mec* type III



SCC*mec* type II



SCC*mec* type IV



Ito et al 2001. AAC
Hiramatsu et al 2001. Trends in Microb.
Oliveira et al. 2001. Microb. Drug Resist.

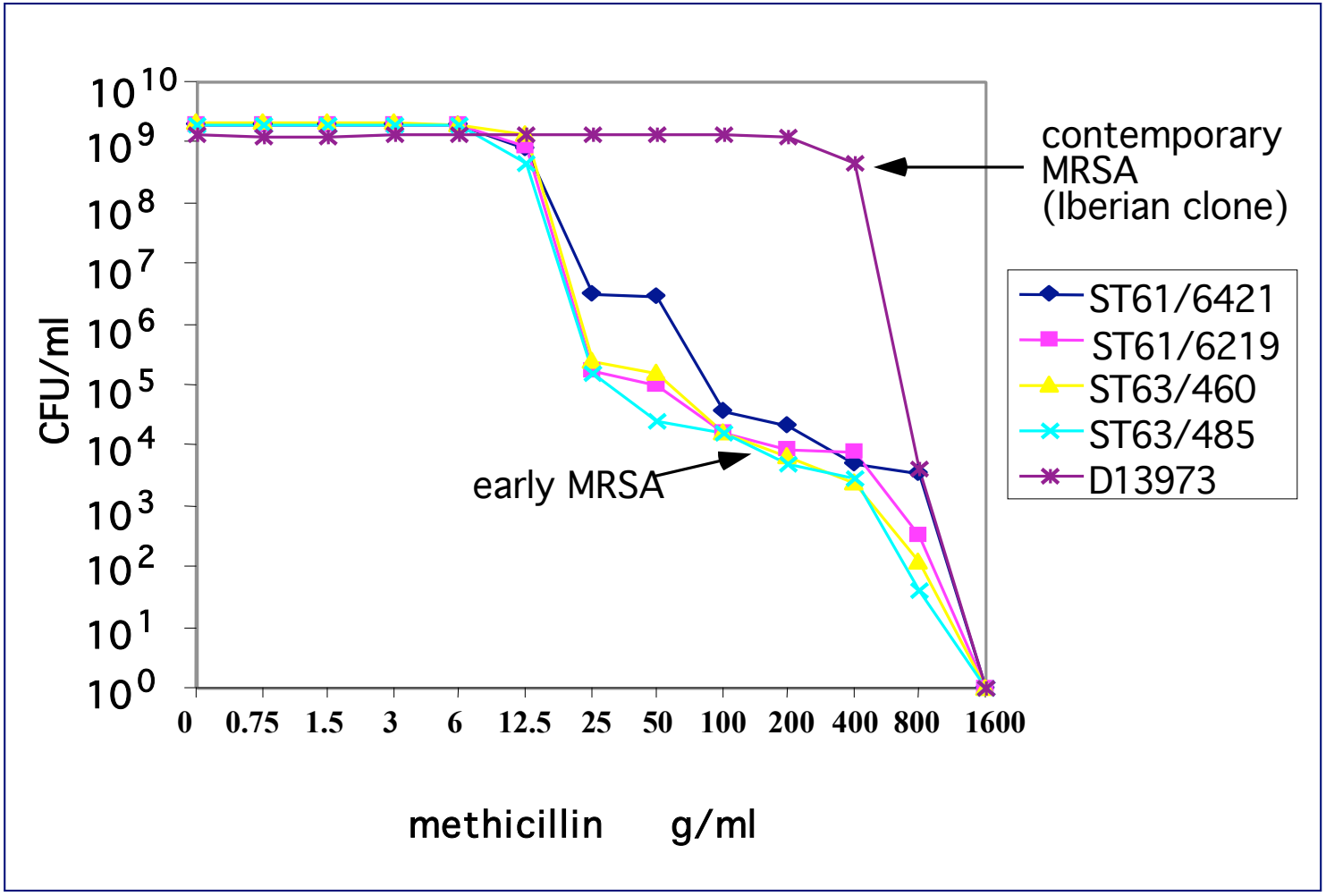


mecA

IS*431*

LJ - chromosomal left junction
RJ - chromosomal right junction

Population Analysis Profile



Grow bugs permissive media, plate on media with increasing antibiotic, count colonies.

Resistance and growth:

- Resistance to vancomycin develops progressively by mutations in multiple genes, what are they?
- Heteroresistance strategy: resistance imposes growth penalty (~2x) thus spontaneously mutate $1{:}10^5$ bugs, fitness cost $1\text{-}10^{-5}$. What genes involved?

.

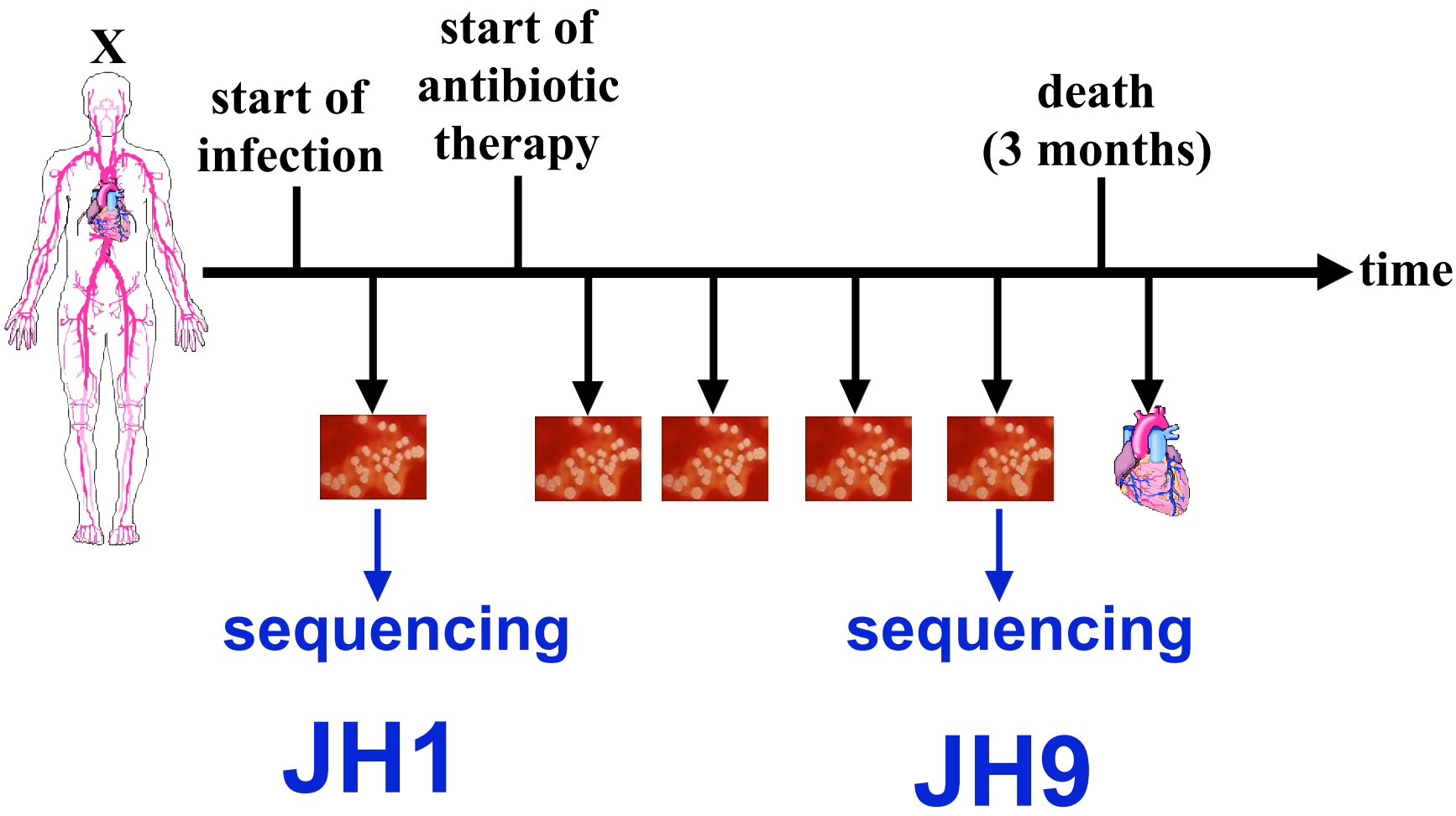
Antibiotic resistance via genome sequencing

1. Intermediate vancomycin resistance multigenetic trait and no obvious ‘resistance’ gene.

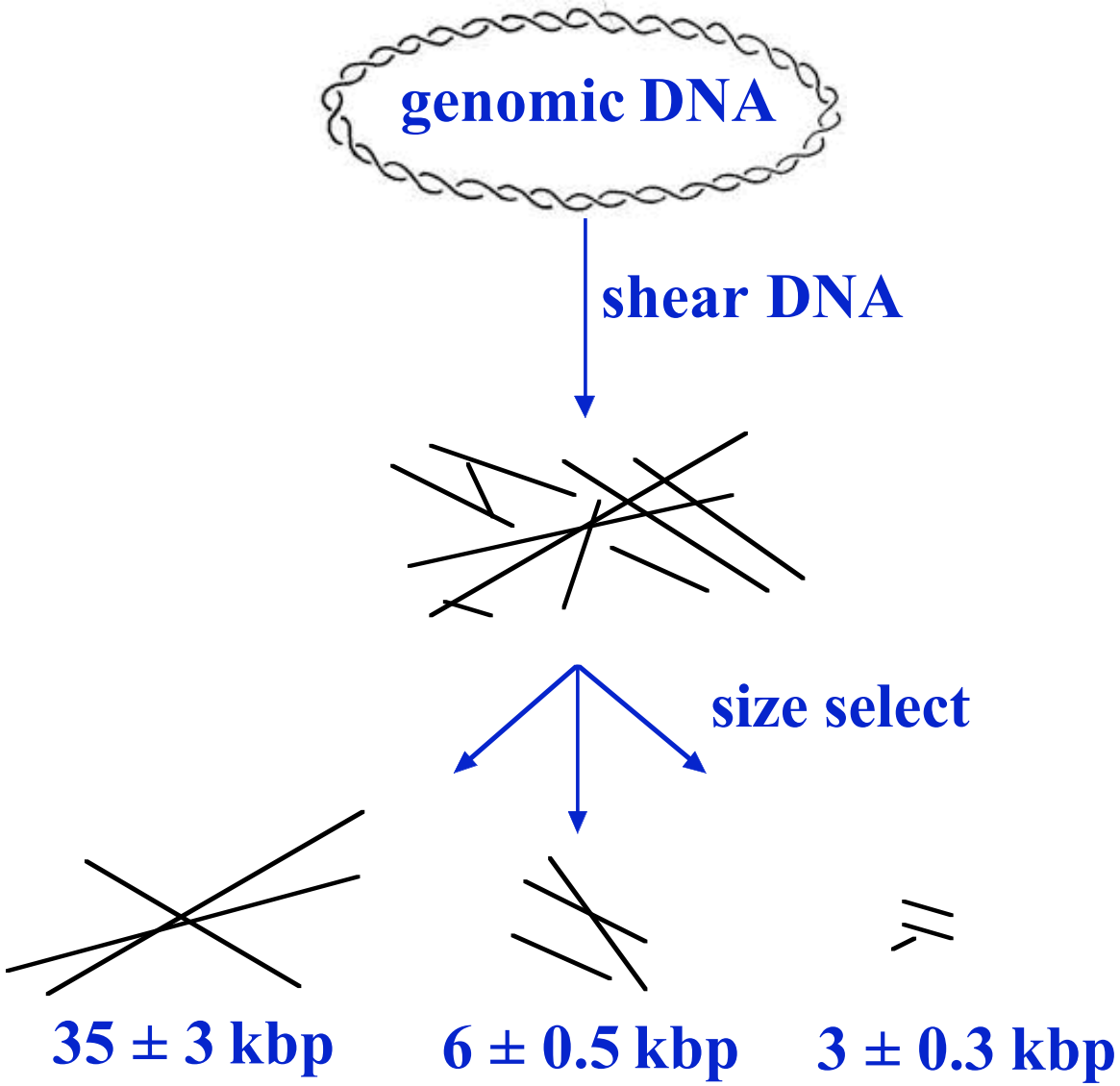
2. What mutations are involved in creating heterotypic resistance population analysis curves.

No lack of S.aureus genomes (9 to date), but none close enough (0.01% - 2% point mutation rate + 10’s kb of novel mobile elements)

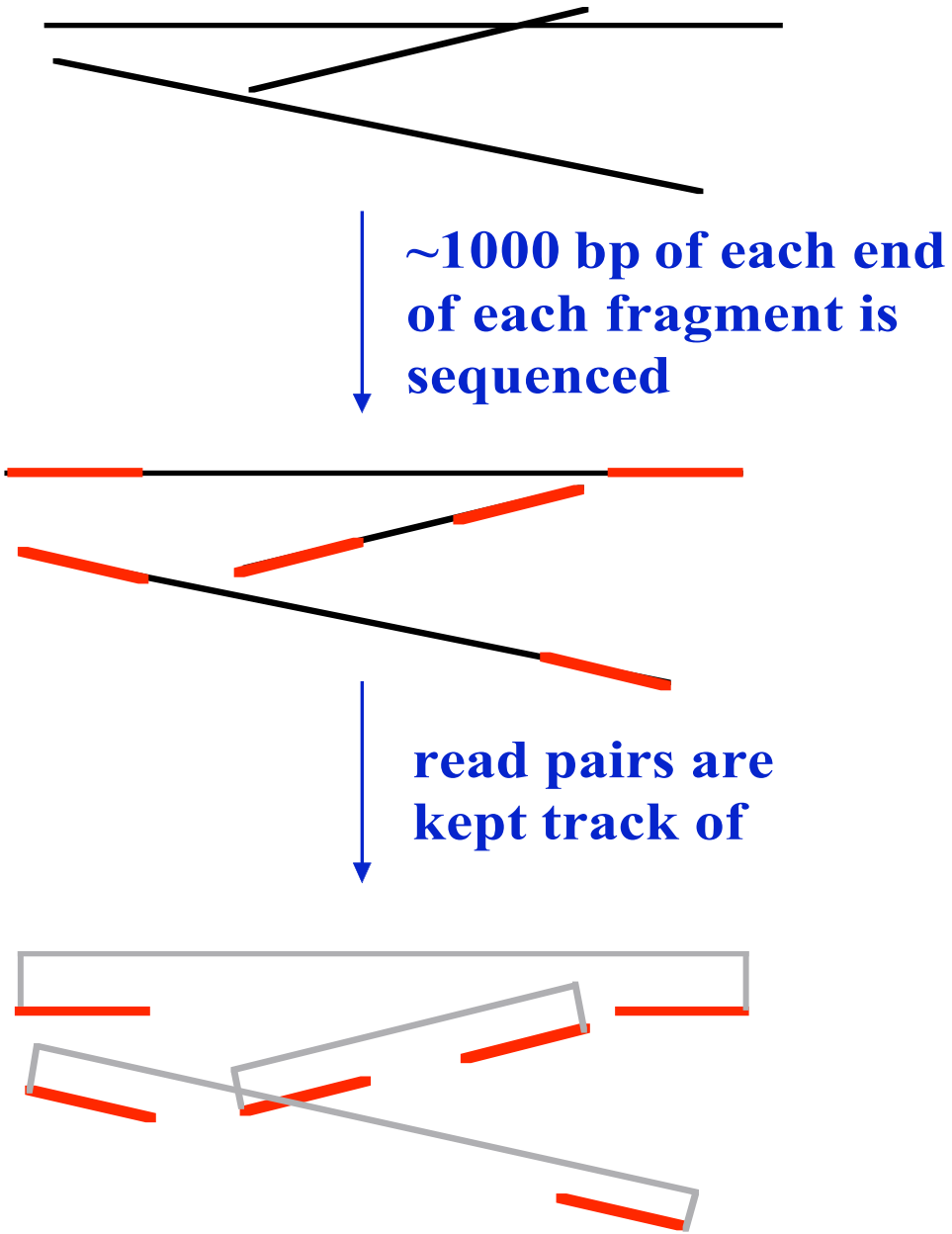
Compare isolates from one patient undergoing vanco therapy of MRSA



Shotgun sequencing I

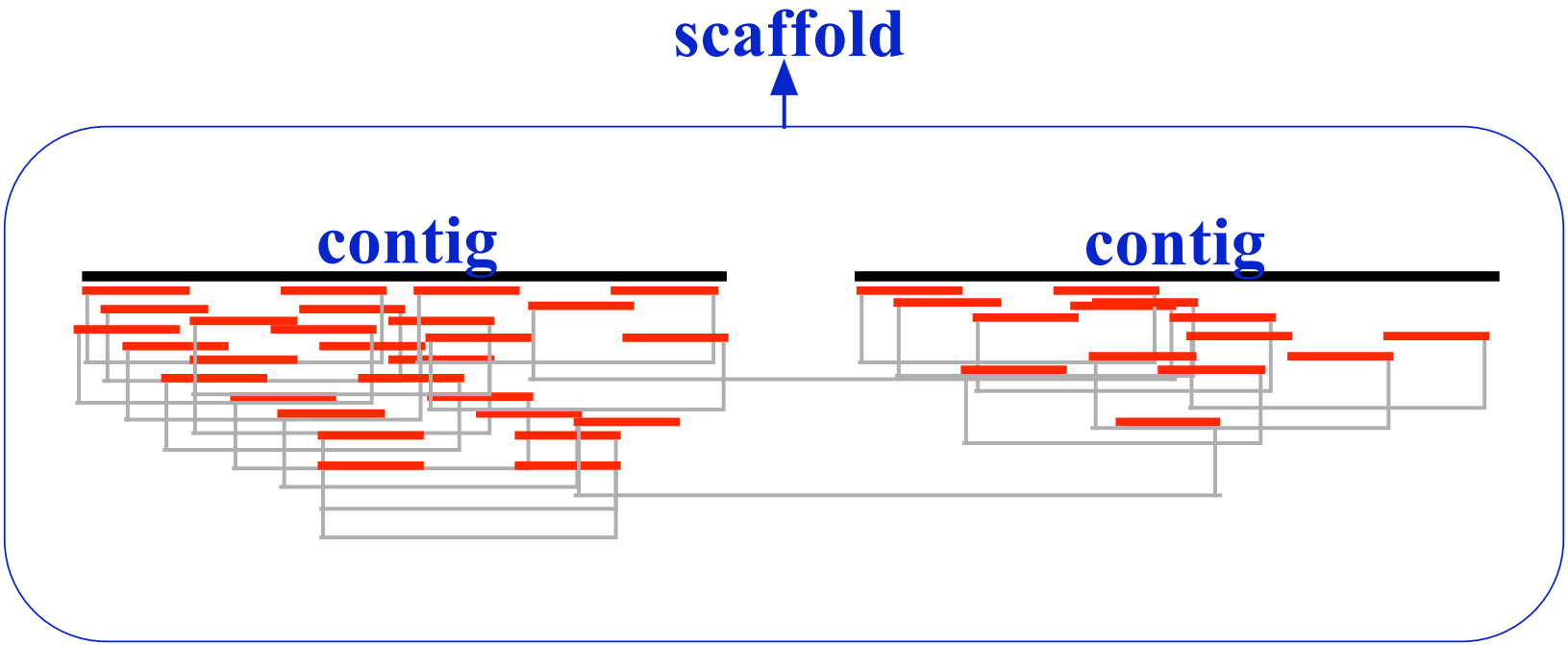


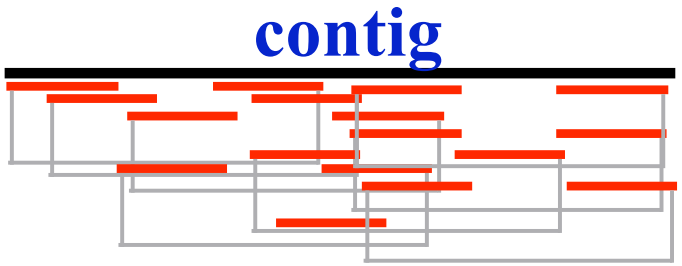
Shotgun sequencing II



JGI

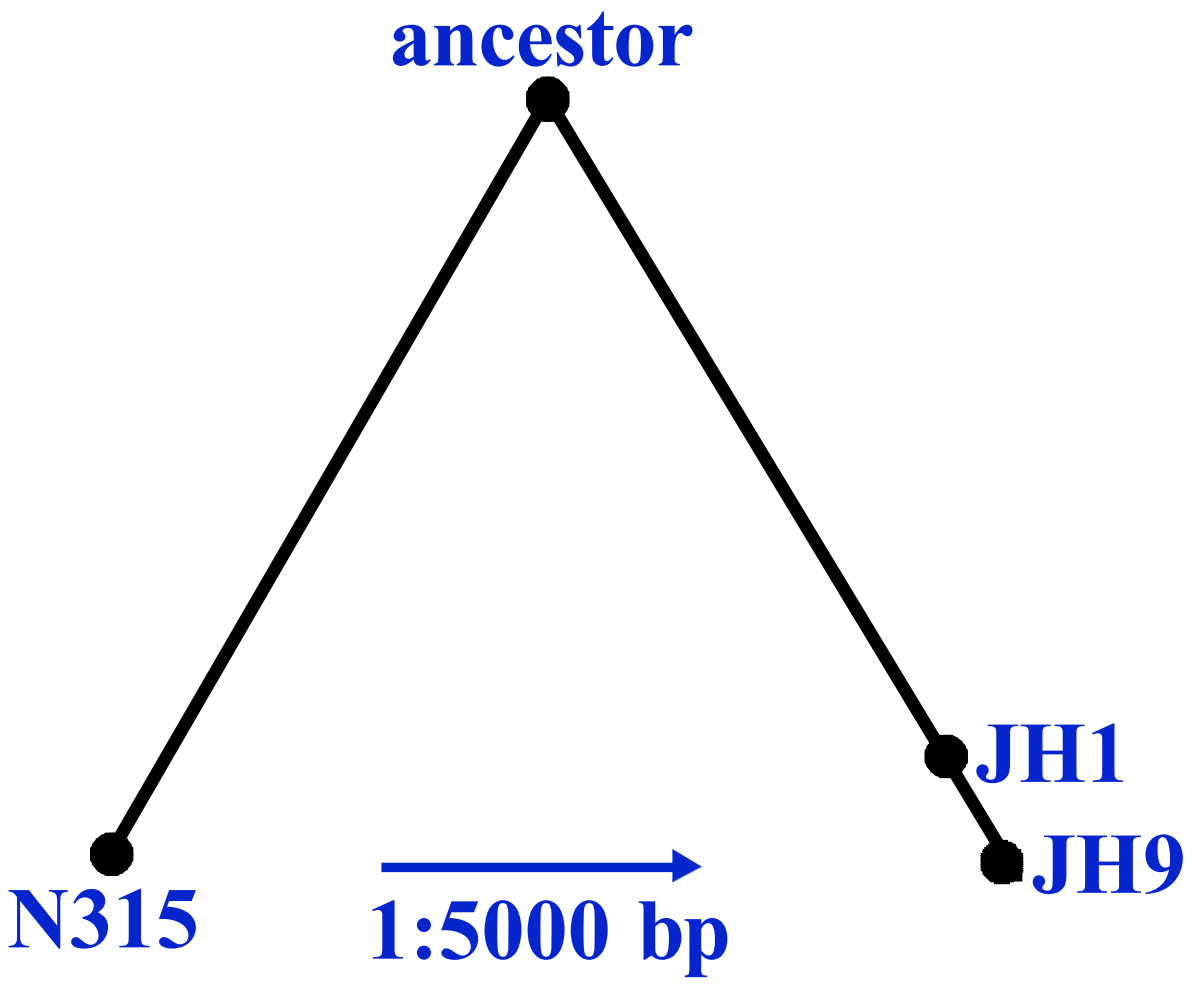
Assembly using Celera assembler



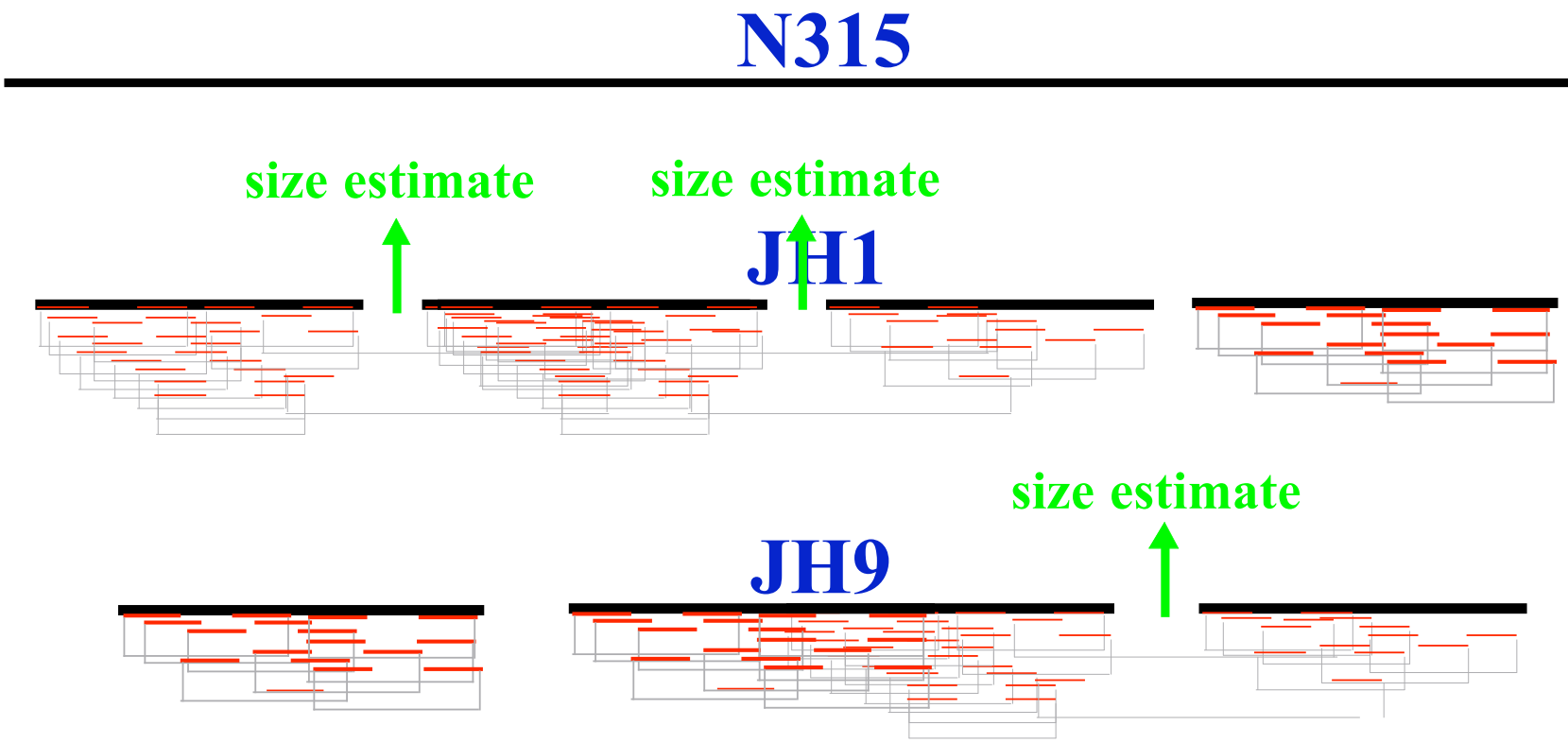


Total genome 3 10^6 bp
~50 contigs/genome

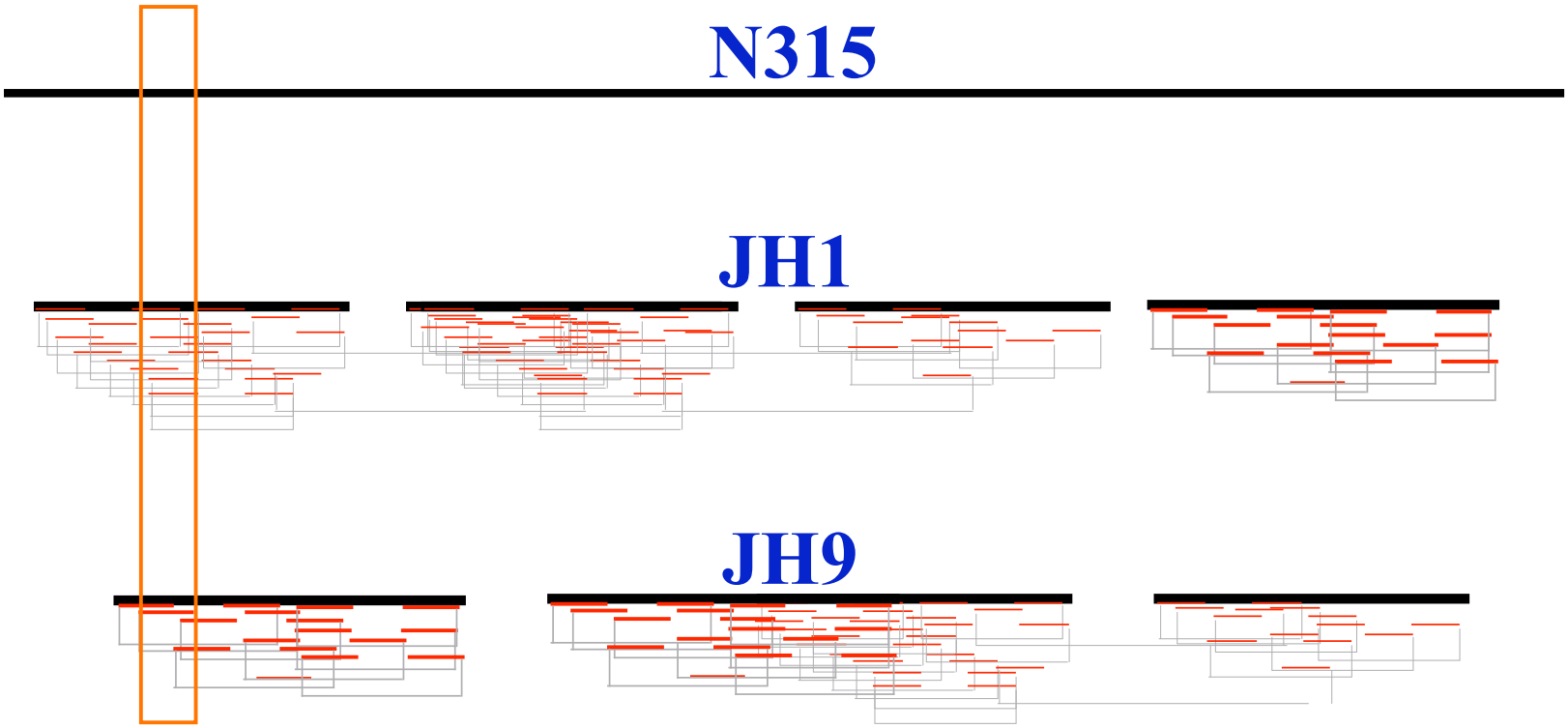
Use of previously sequenced N315



Multialignment I



Read errors and quality values

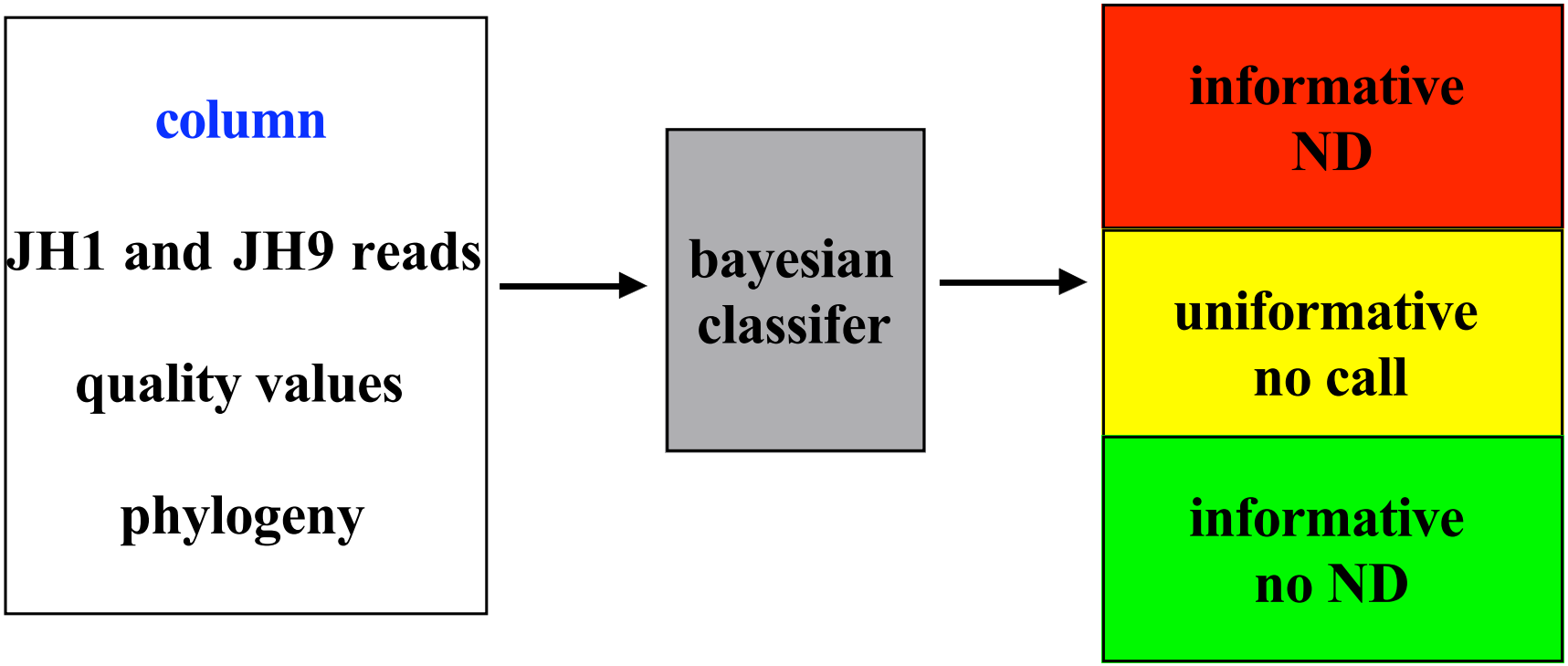


Read errors and quality values

N315		**GATTCGA**
	read 1	**GATTCGA**
	read 2	**GATTCGA**
JH1	**read 3**	**GACTCGA**
	read 4	**GATTCGA**
	read 5	**GATTCGA**
	read 1	**GATTCCA**
JH9	**read 2**	**GATTCGA**
	read 3	**GATTCCA**

$$E = 10^{-Q/10}$$

Bayesian classifier

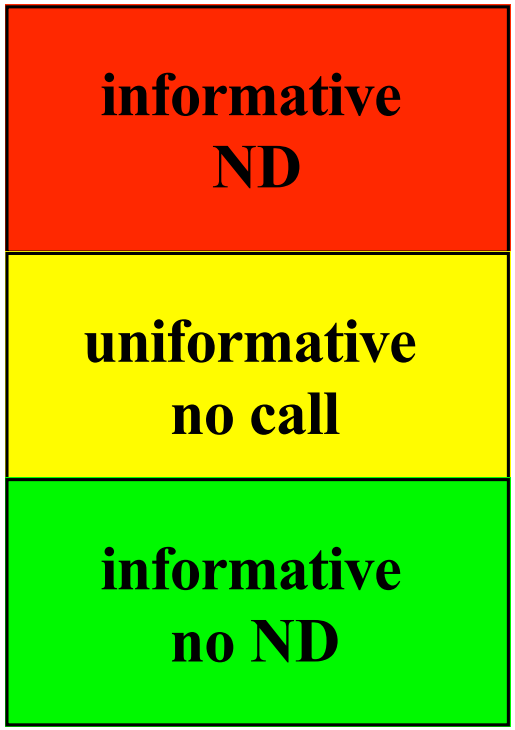


Validation of Bayesian classifier by PCR sequencing

35bases mutated

6% can't tell

94% no change



PCR sequenced

3 10^6 bp genome

Summary of Assembly and mutations

- 2.9Mb genomes, ~8x coverage, ~60 contigs, ~2% sequence in gaps (30kb plasmid ~50x one contig)
- Changes: N315 vs JH1 about 120kb new sequence in mobile elements, 1:5000 point mutation rate
- Changes: JH1 vs JH9 Call point mutations on 94% of bases (eg 6% = 2% +2% gaps + 2% low coverage) -> 34 total. No larger elements, all gaps consistent with no change

Temporal order of mutations

Isolate	JH1	JH2	JH5	JH6	JH9	JH14
Day	1	63	74	79	86	90
Locus	Blood					Heart
blaR1						
rpoB						
rpoC						
SA1129						
SA1702						
SA1249						
SA0019						
SAS014						
SA0582						
SA0980						
SA1659						
SA1843						
SA2094						
SA2 125						
SA2320						
SAP007						
SA0171						
SAtRNA34						
other loci					+12	

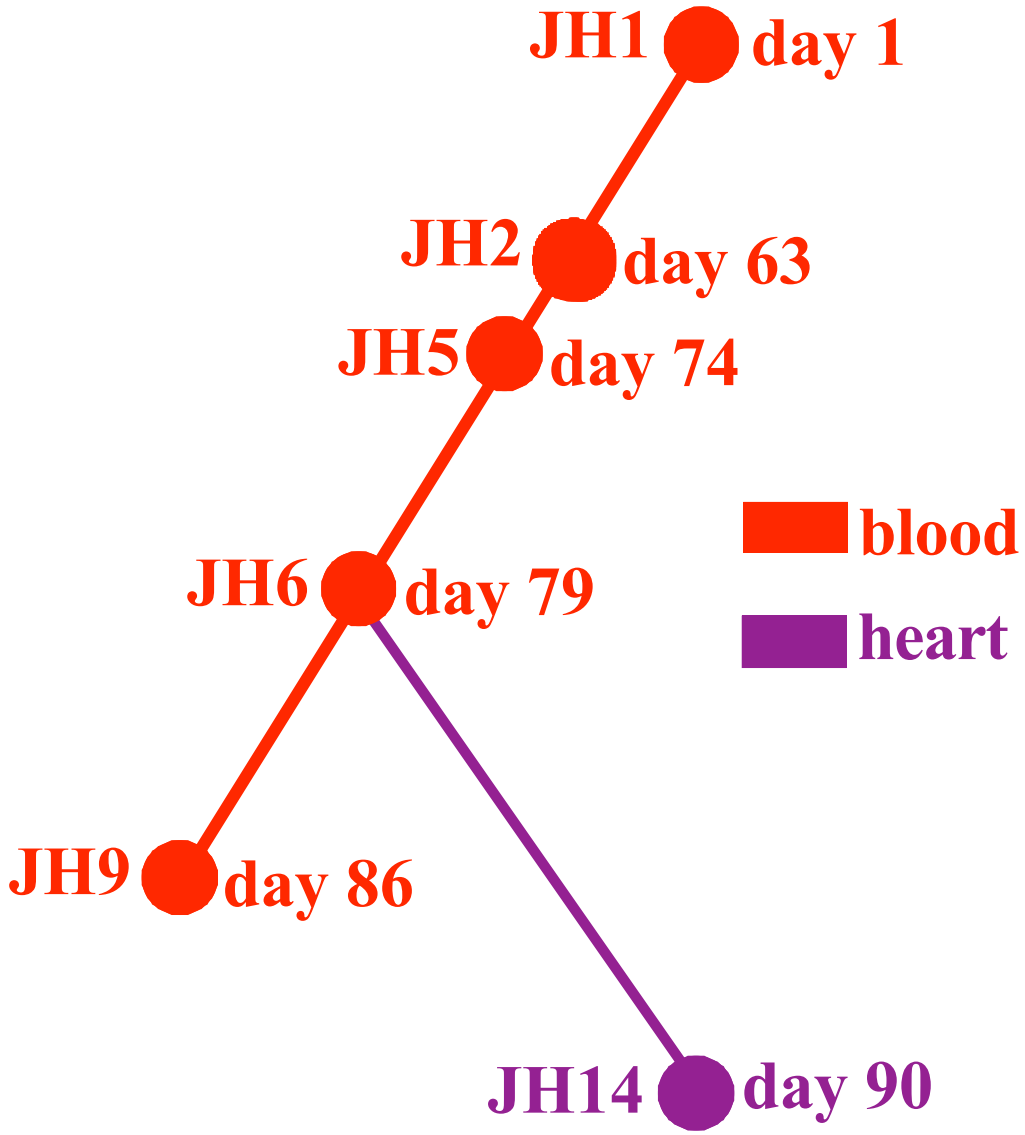
cccccccc +-c
hypermutable

Wu, Yanjiao

Establishment of heart infection

Isolate	JH1	JH2	JH5	JH6	JH9	JH14
Day	1	63	74	79	86	90
Locus	Blood					Heart
blaR1						
rpoB						
rpoC						
SA1129						
SA1702						
SA1249						
SA0019						
SAS014						
SA0582						
SA0980						
SA1659						
SA1843						
SA2094						
SA2 125						
SA2320						
SAP007						
SA0171						
SAtRNA34						
other loci					+12	

Establishment of heart infection



Determination of resistance levels

MIC (µg/ml)	**Vancomycin**	1.0	4.0	6.0	8.0	8.0	8.0
	Daptomycin	0.01	0.05	0.05	1.0	1.0	1.0
	Rifampin	0.012	16	16	16	16	16
	Oxacillin	0.75	25	0.75	1.5	0.75	1.5
Isolate		JH1	JH2	JH5	JH6	JH9	JH14
Day		1	63	74	79	86	90
Locus		Blood					Heart
blaR1							
rpoB							
rpoC							
SA1129							
SA1702							
SA1249							
SA0019							
SAS014							
SA0582							
SA0980							
SA1659							
SA1843							
SA2094							
SA2125							
SA2320							
SAP007							
SA0171							
SAtRNA34							
other loci						+12	

primary treatment

use discovered from genome

characteristic rif. mutations

Mutations not involved in resistance

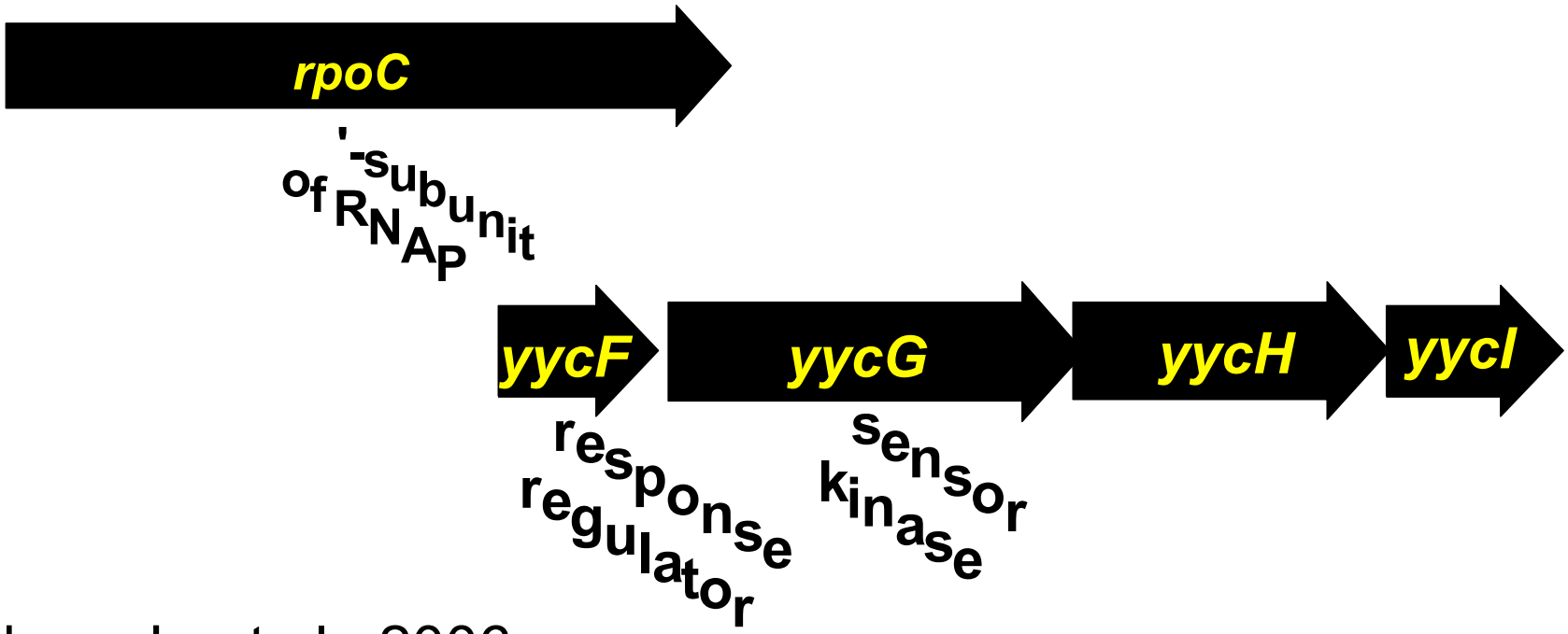
Potential resistance determinants

Vancomycin
SA1702 (in operon with *vraSR*)
agrC
yycH
nagB
Daptomycin
yycH
rpoC
Rifampin
rpoB
ß-lactams
blaR1

All signaling pathways -> expression of many genes change

Resistance to new Ab, daptomycin, involves similar mutations

vitro evolution created strain with miss-sense mutantions in rpoC, rpoB, yycG, (+2 other genes) with reduced daptomycin susceptibility



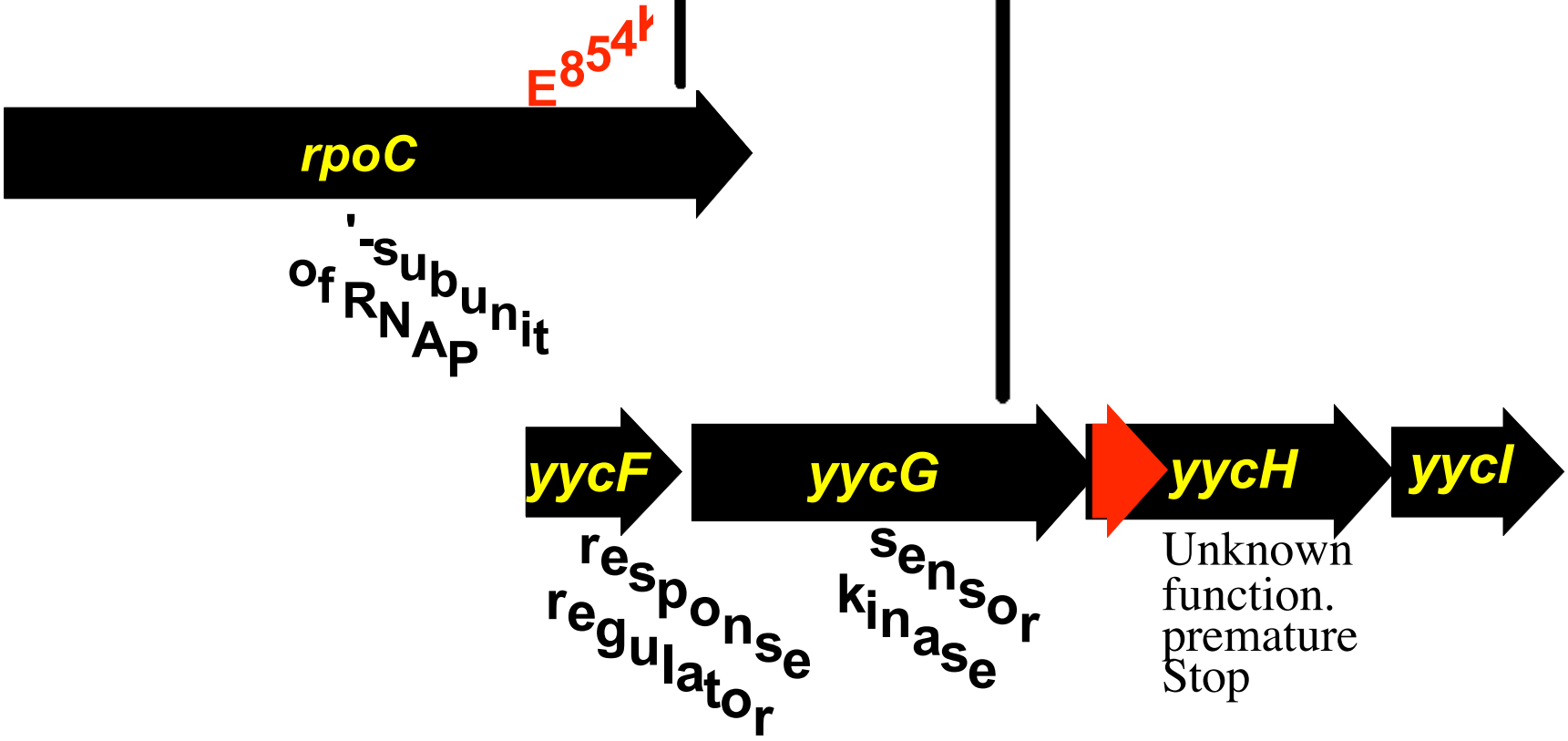
Friedman L. et al., 2006

Cross resistance

- Daptomycin approved in 2003 for vanco resistent S.aureus (2y after patient died, and not used)
- Common mutations suggest cross resistance between daptomycin and vancomycin.
- JB Patel etal 2006, CDC report documents cross resistance between vancomycin and daptomycin
- Went back and checked our isolates….

Jumps in daptomycin resistance coincide with mutations in rpoC and yyc operon

Daptomycin MIC (µg/ml)	0.01	0.05	0.05	1.0	1.0	1.0
Isolate	JH1	JH2	JH5	H6	JH9	JH14



What's next

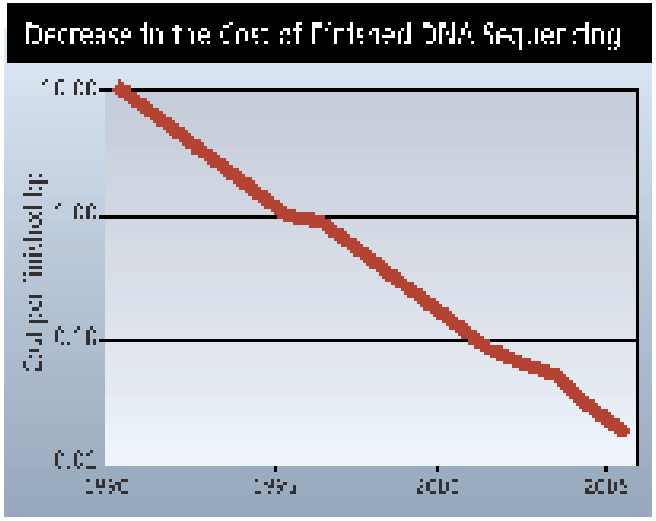
Lots more sequencing…

•Is there one path to intermediate vancomycin resistance or many?? Sequence strain collection, study other patients.

•What mutations are responsible for converting heteroresistant MRSA to high homogeneous resistance (untreatable)??

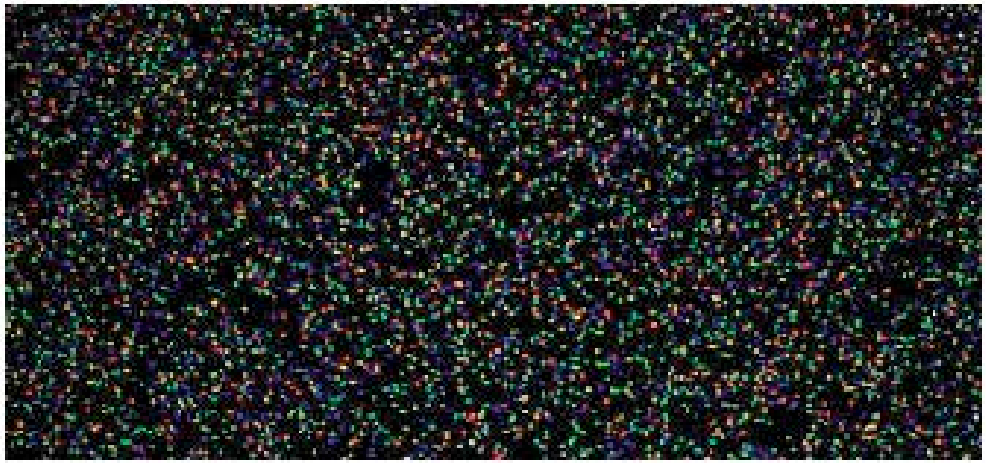
Previous project took 1.5 years

But what about sequencing



- 2004 VOL 31 SCIENCE www.sciencemag.org
Published by AAAS

1990-2005 costs decrease by 1000x



GENE SEQUENCING

The Race for the $1000 Genome

Fast, cheap genetic analyses will soon become a reality, and the consequences—good and bad—will affect everybody

MARCO ISLAND, FLORIDA—Computers aren't the only machines getting faster and cheaper every time

sign (Harvard University sequencing pioneer George Church.

2005-6, 1 bacteria ~ $\$10^4$, 1 human ~ $\$2\text{-}10\text{x}10^6$, can it go to $1000

Scanning for mutations via arrays

Genome-Wide Detection of Polymorphisms at Nucleotide Resolution with a Single DNA Microarray

David Gresham,[1,2*] Douglas M. Ruderfer,[1,3] Stephen C. Pratt,[1,3] Joseph Schacherer,[1,3] Maitreya J. Dunham,[1] David Botstein,[1,2] Leonid Kruglyak[1,3*]

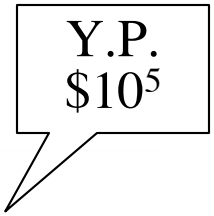
A central challenge of genomics is to detect, simply and inexpensively, all differences in sequence among the genomes of individual members of a species. We devised a system to detect all single-nucleotide differences between genomes with the use of data from a single hybridization to a whole-genome DNA microarray. This allowed us to detect a variety of spontaneous single–base pair substitutions, insertions, and deletions, and most (>90%) of the ~30,000 known single-nucleotide polymorphisms between two Saccharomyces cerevisiae strains. We applied this approach to elucidate the genetic basis of phenotypic variants and to identify the small number of single–base pair changes accumulated during experimental evolution of yeast.

Thoughts of Chairman Mao



"Let a hundred flowers bloom,
let one hundred schools of thought contend"

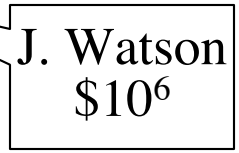
















Conclusions

•Microevolution + sequencing much more informative than expression arrays (pt mutations in a few pathways vs 200 genes change expression)
•Attractive approach to decipher heteroresistance MRSA, are same genes, operons, pathways, mutated in all isolates??

•Solexa: $400k per machine (300k euros), 3k reagents + 1k overhead per run, multiplex 8 samples, length 30-35 reads (good on poly-N), 99.7% accuracy 1-1.2x10^9 bp -> $600/bacteria (30x)

Credits

Michael Mwangi (Cornell Physics PhD 2006)

Alex Tomasz (Rockefeller, Shang-Wei Wu, Zhou Yanjiao, Krzysztof Sieradzki, Herminia de Lencastre)

Joint Genome Inst (P. Richards, David Bruce, Eddy Rubin..)

Gene Myers

References

General:

"Antibiotics, Actions, Origins, Resistance", C Walsh

"Bacterial Pathogenesis..", Salyers&Whitt

Recent Whole Genome Evolution Papers

M.Olson, PNAS, Pseudomonas in cystic fibrosis

B. Palsson, Nature Genetics Ecoli

WASH YOUR HANDS!

(some of) my encounters with Vinay Ambegaokar

77/78 Postdoc with Vinay at LASSP Cornell

nonequilibrium superconductivity *cross-country skiing*

81 Nordita-Landau Institute meeting in Sweden

quantum dissipation due to quasiparticle tunneling

84 Workshop on ‘Quantum Dissipation’ at ITP Santa Barbara

qu. diss. ctd., notions of single-electron effects

86-90s A. v. Humboldt Award for Vinay: visits to Albert Schmid in Karlsruhe

ballet visit and ice storm



Solid State Qubits and Decoherence

Gerd Schön	*Karlsruhe*	http://www.tfp.uni-karlsruhe.de/

work with:

Alexander Shnirman	*Karlsruhe*
Yuriy Makhlin	*Landau Institute*
Pablo San-Jose	*Karlsruhe*
Gergely Zarand	*Budapest*

- **Josephson qubits**
- **Noise and decoherence**
- **Geometric dephasing of spins in quantum dots**



Universität Karlsruhe (TH)
Research University · founded 1825

CFN DFG

KIT Karlsruhe Institute of Technology

Josephson Charge Qubit

theory: Caldeira, Leggett (1981)
Ambegaokar, Eckern, G.S. (1982)
Shnirman, Makhlin, G.S. (1997+)

expt: Nakamura, Pashkin, Tsai (NEC 1999)
Devoret, Esteve, … (Saclay 2002)
….

$$H = E_C (2n - \frac{C_g V_g}{e})^2 - E_J \cos(\pi \frac{\Phi_x}{\Phi_0}) \cos\theta$$

2 degrees of freedom

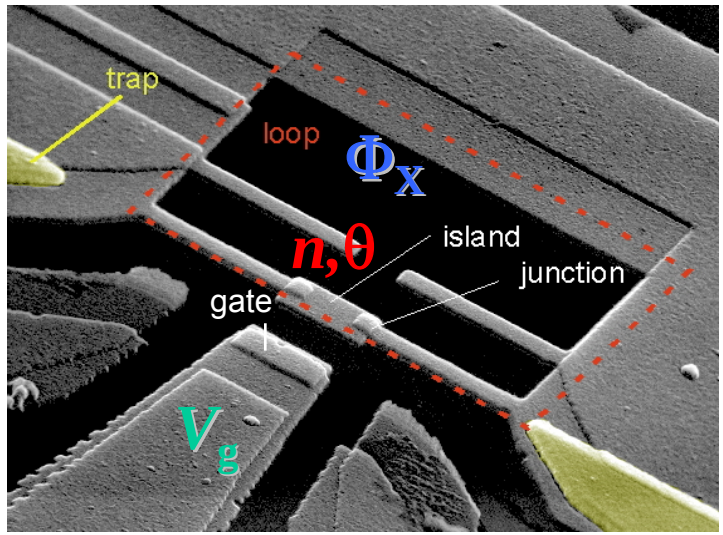
$[n, \theta] = -i$ charge and phase

2 energy scales E_C, E_J

charging energy, Josephson coupling

2 control fields: V_g and Φ_x

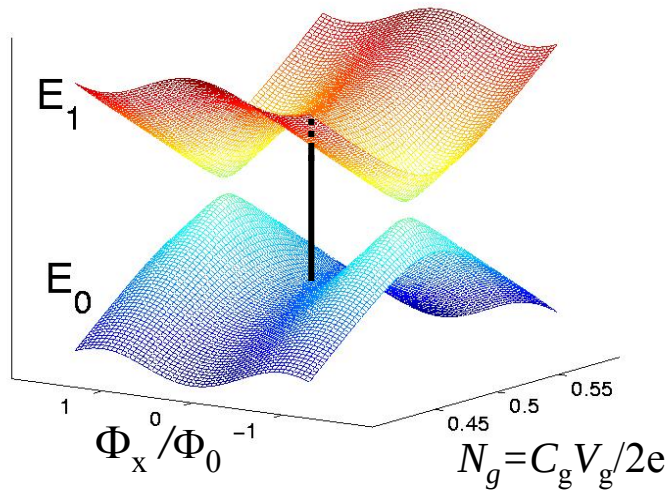
gate voltage, flux



2 states only, e.g. for $E_C \gg E_J$

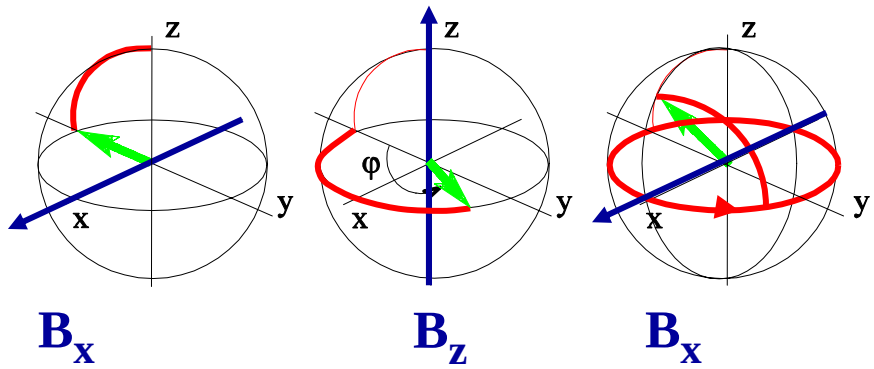
$$H = -\frac{1}{2}\Delta E_{ch}(V_g)\,\sigma_z - \frac{1}{2}E_J(\Phi_x)\,\sigma_x$$

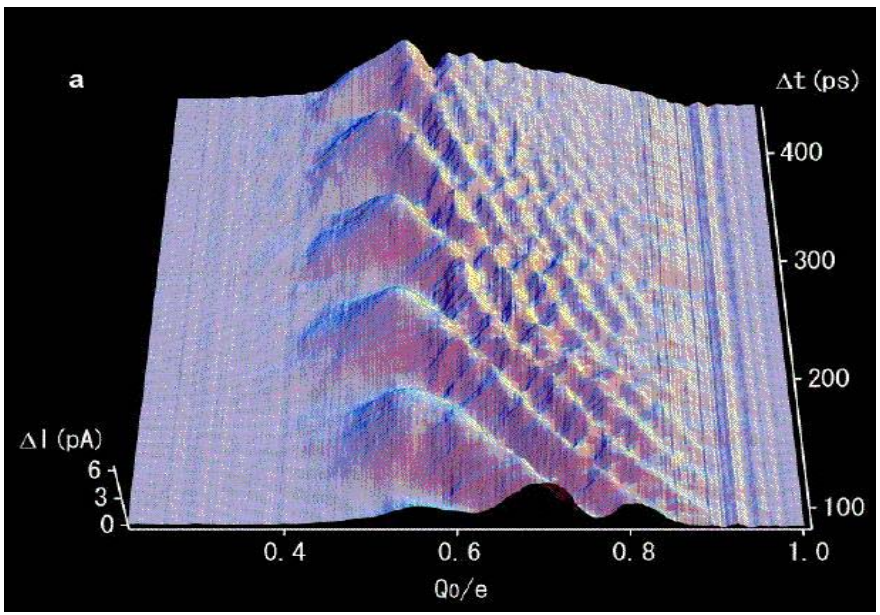
Two basis/logic states differ by
one Cooper-pair charge n and $n+1$ on island

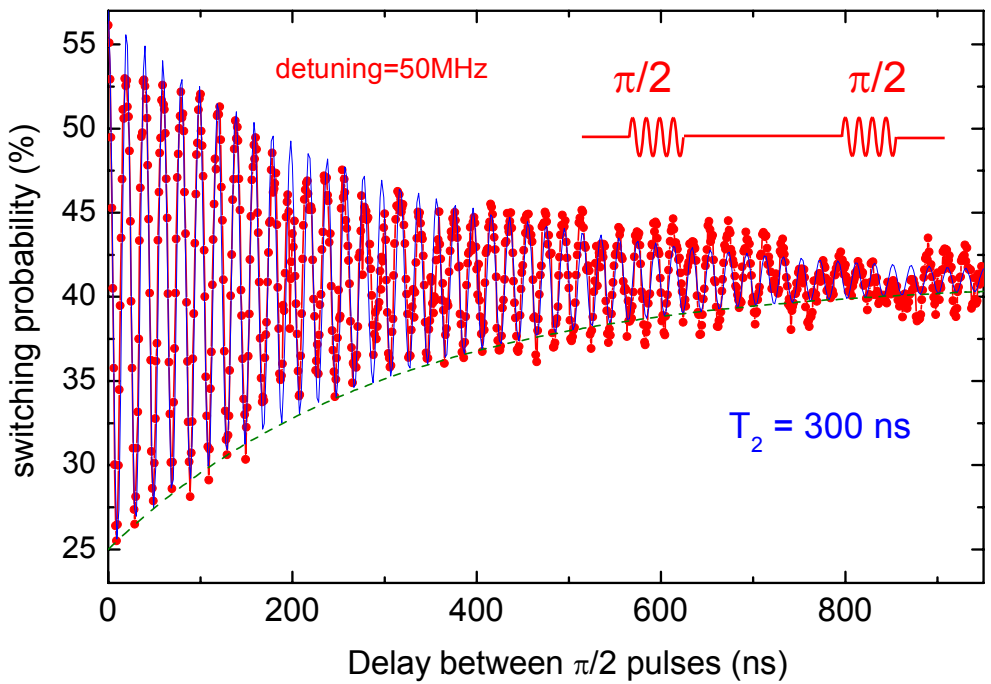


Quantum coherent oscillations
(Ramsey fringes)

$$H = -\frac{1}{2}\Delta E_{ch}(V_g)\,\sigma_z - \frac{1}{2}E_J(\Phi_x)\,\sigma_x$$



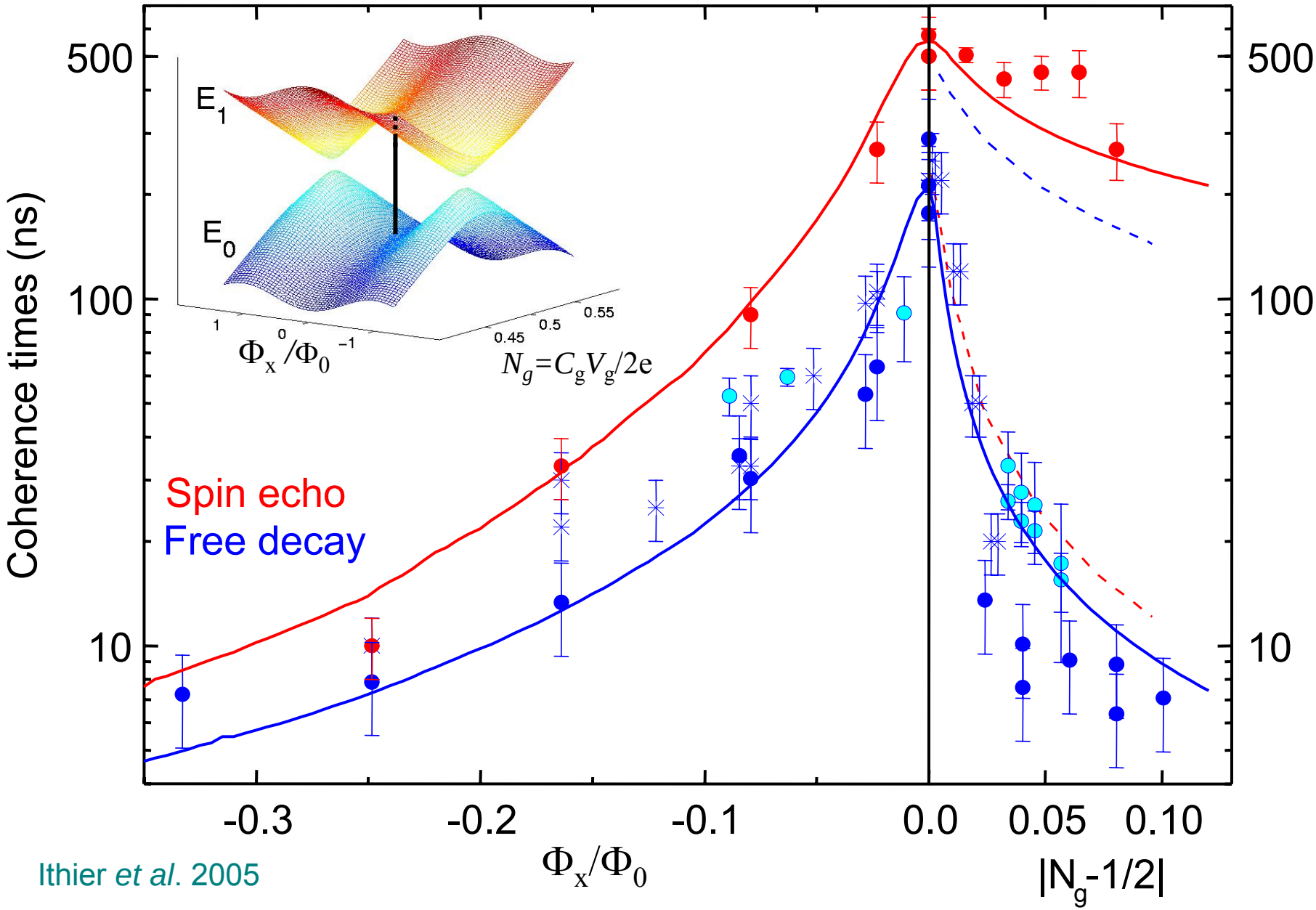




Nakamura *et al*. (Nature 99): τ_φ = 5 nsec

Vion *et al*. (Science 02) τ_φ = 300 nsec – 5 μsec

Optimum-point strategy for noise reduction



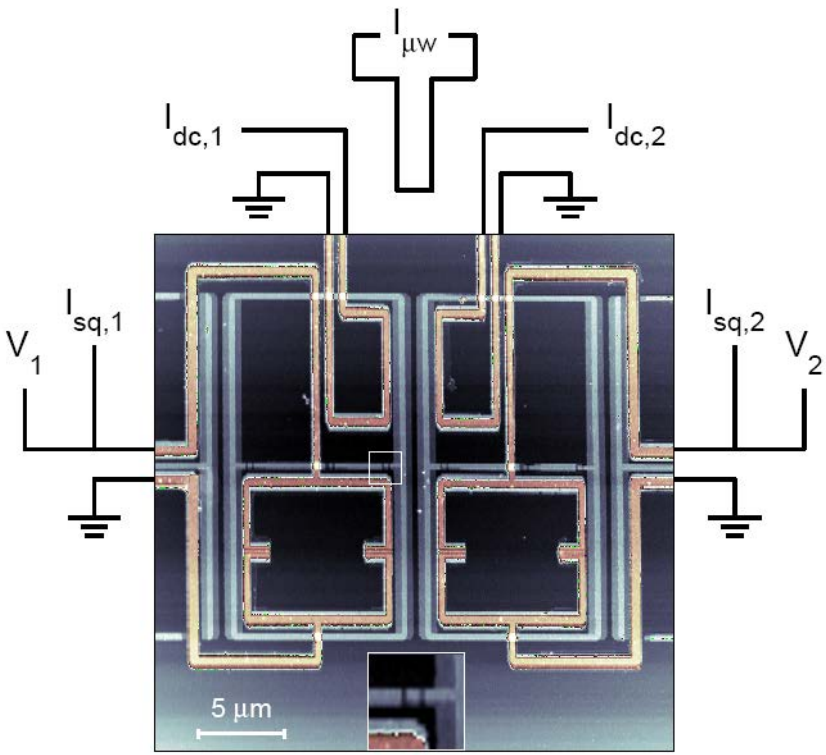
Ithier *et al*. 2005

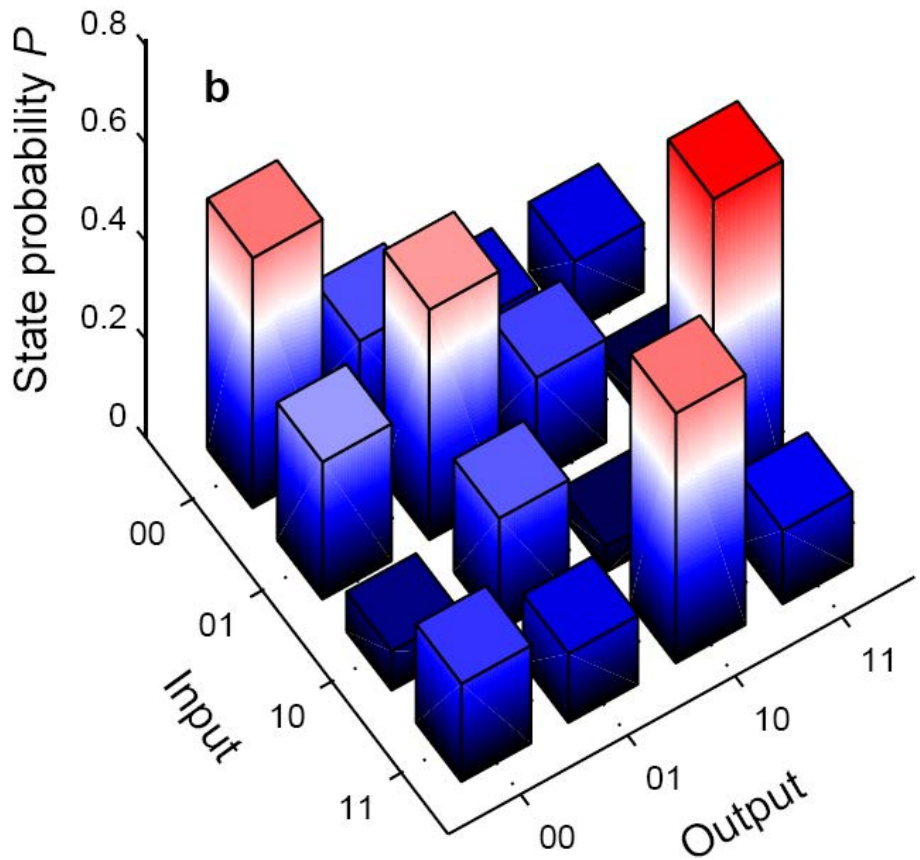
nature Vol 447|14 June 2007|doi:10.1038/nature05896

LETTERS

Demonstration of controlled-NOT quantum gates on a pair of superconducting quantum bits

J. H. Plantenberg[1], P. C. de Groot[1], C. J. P. M. Harmans[1] & J. E. Mooij[1]





Noise and Decoherence

Sources of noise

- noise from control and measurement circuit, $Z(\omega)$
- background charge fluctuations
- …

Properties of noise

- spectrum: Ohmic (white), *1/f*, …
- Gaussian, 2-level systems, …

$$S_X(\omega) = \frac{1}{2}\int dt \left\langle \{X(t), X(0)\}_+ \right\rangle e^{i\omega t}$$

$$\propto \omega \coth \frac{\hbar\omega}{2k_B T}, \quad 1/\omega, \ldots$$

- coupling:

$$H = -\frac{1}{2}\left(\Delta E\,\tau_z + X_{\parallel}\tau_z + X_{\perp}\tau_x + \lambda X_{\parallel}^2\,\tau_z\right) + H_{\text{bath}}$$

longitudinal – transverse – quadratic (longitudinal) …

Linear coupling, regular power spectrum

$$H = -\frac{1}{2}\left(\Delta E\,\tau_z + X\cos\eta\,\tau_z + X\sin\eta\,\tau_x\right) + H_{Bath}$$

Golden rule (Bloch – Redfield) $\Rightarrow$ exponential decay

(Leggett *et al*. 87, Weiss 99)

$$|\rho_{ij}(t)| \propto e^{-\Gamma t}$$

$$\frac{1}{T_1} = \Gamma_{\text{rel}} = \frac{1}{2} S_X(\omega = \Delta E)\sin^2\eta$$

$$\frac{1}{T_2} = \Gamma_{\varphi} = \frac{1}{2}\frac{1}{T_1} + \underbrace{\frac{1}{2} S_X(\omega \approx 0)\cos^2\eta}_{\text{pure dephasing: } \Gamma_{\varphi}^{*}}$$

Nyquist noise due to R

$$S_{\delta V}(\omega) = \hbar\omega\, R \coth\frac{\hbar\omega}{2k_B T}$$

$$\Rightarrow \quad \Gamma_{\text{rel}} \propto (e^2/h)\, R\, \Delta E$$

$$\Gamma_{\varphi}^{*} \propto (e^2/h)\, R\, k_B T$$

Dephasing due to 1/f noise, nonlinear coupling, … ?

longitudinal linear coupling, 1/*f* noise

$$H = -\frac{1}{2}(\Delta E + X)\tau_z + H_{Bath}$$

$$S_X(\omega) = \frac{E_{1/f}^2}{|\omega|}$$

Golden rule $\Gamma_\varphi^* = \frac{1}{2} S_X(\omega=0)$ fails for $1/f$ noise, where $S_X(\omega) = \to \infty$ for $\omega \to 0$

beyond Golden rule:

$$|\rho_{01}(t)| = \exp\left(-\frac{1}{2}\int \frac{d\omega}{2\pi} S_X(\omega) \frac{\sin^2(\omega t/2)}{(\omega/2)^2} \right)$$

$$|\rho_{01}(t)| = \exp\left(-\frac{E_{1/f}^2}{2\pi} t^2 \ln|\omega_{ir} t| \right)$$

Non-exponential decay of coherence

Cottet *et al*. (01)

$$\Gamma_\varphi^* \propto E_{1/f}$$

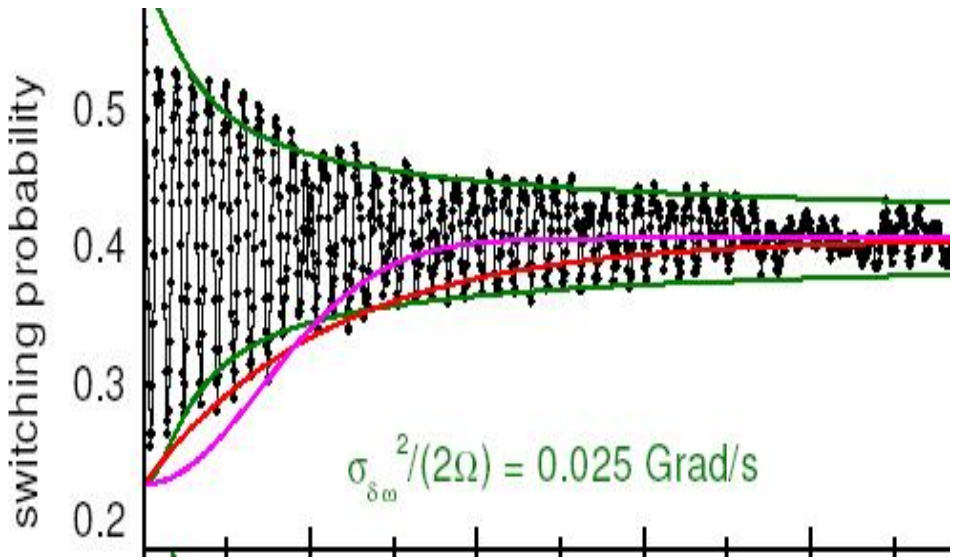
At optimal point: Quadratic longitudinal 1/f noise

$$H = -\frac{1}{2}\left(\Delta E + \lambda X^2\right)\tau_z \qquad S_X(\omega) = \frac{E_{1/f}^2}{|\omega|}$$

$$|\rho_{01}(t)| = \left[1 + i\frac{2}{\pi}\Gamma_{1/f}\, t\, \ln(\omega_{ir} t)\right]^{-1/2}$$

Power law decay $\Gamma_{1/f} = \lambda E_{1/f}^2$

Shnirman *et al*. (03,05)



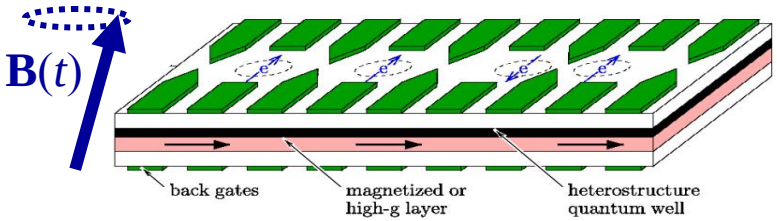
$e^{-\Gamma^2 t^2}$

$e^{-\Gamma t}$

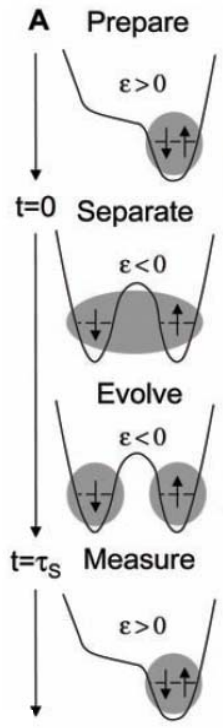
$1/|\sqrt{1 + i\Gamma t}|$

Ithier *et al*., (2005)

Spin qubits: single electrons in quantum dots

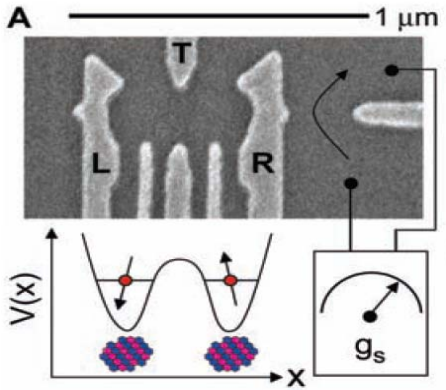


Loss, DiVincenzo (98)



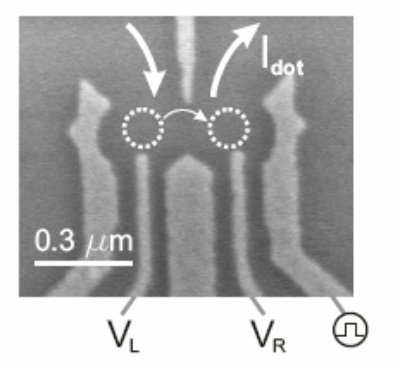
Coherent manipulation of coupled electron spins in semiconductor quantum dots

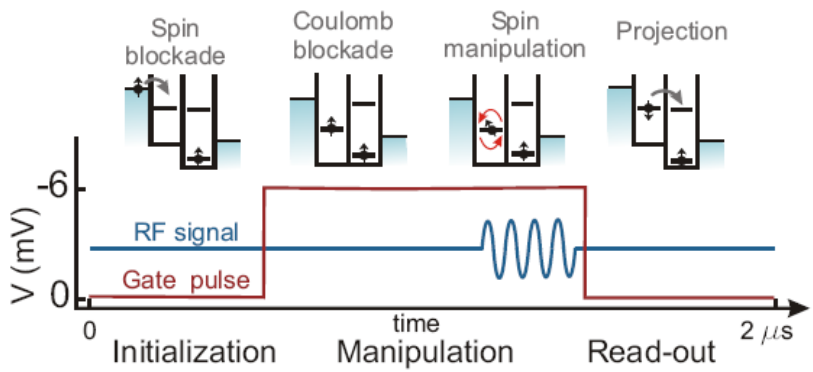
Petta *et al*. (2005)



Driven coherent oscillations of a single electron spin in a quantum dot

Koppens, …, Vandersypen (2006)



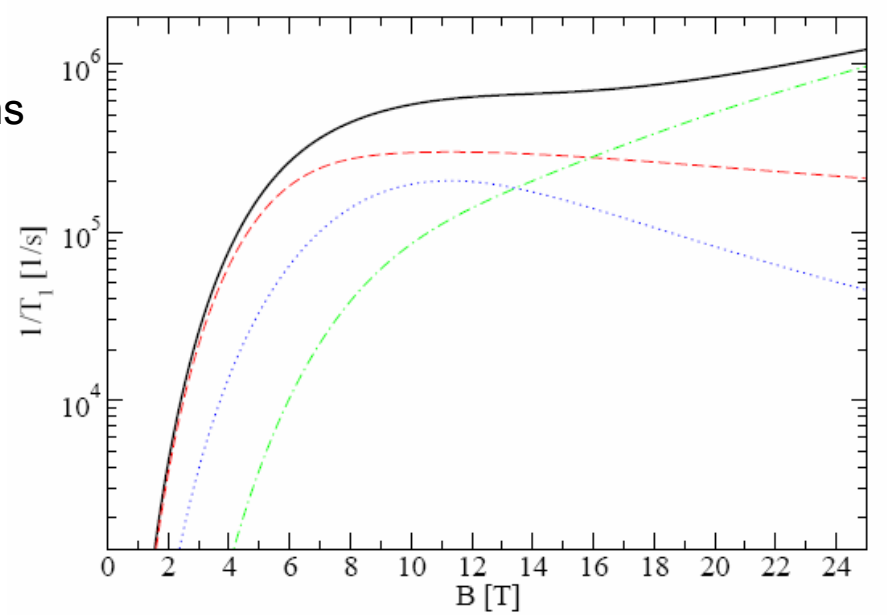


Decoherence of spins in quantum dots

- potentially long phase coherence time!
- fluctuations, e.g., due to piezoelectric phonons
- couple via spin-orbit interaction to spin
- + broken time reversal symmetry

  $\Rightarrow$ decoherence
- vanishing decoherence for $\vec{B} = 0$

Khaetskii, Nazarov (2001)
Golovach, Khaetskii, Loss (2004)
Stano, Fabian (2005)



What is spin decoherence at $\vec{B} = 0$?

We find:

- Nonvanishing decoherence at $\vec{B} = 0$
- due to geometric origin (Berry phase)
- requires two independent sources of fluctuations
- Ohmic fluctuations dominate at low B

San-Jose, Zarand, Shnirman, G.S. (2006)
arXiv:0704.2974

Semiclassical description of spin geometric phase

A: Spin-orbit interaction

↔ momentum dependent 'magnetic field'

$$H_{\rm so} = \frac{\hbar}{2}\,\vec{\sigma}\cdot\vec{B}_{\rm so}$$

$$\vec{B}_{\rm so} = \alpha(p_y,-p_x,0) + \beta(-p_x,p_y,0)$$

Rashba Dresselhaus

B: Semiclassical picture:

electron moves distance *dr* in time *dt*

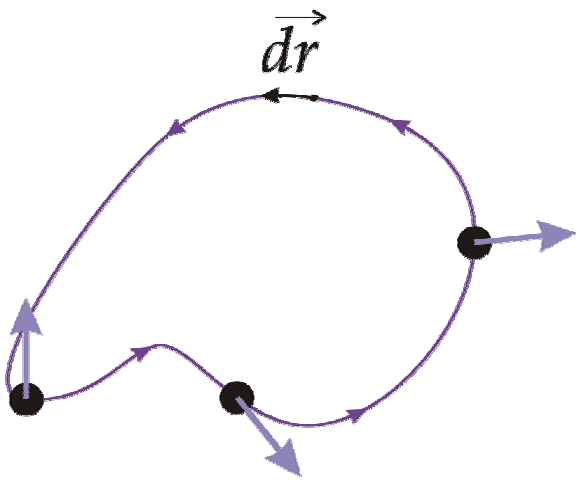
⇒ spin transforms by *U[dr]*, independent of *dt* ('geometric')

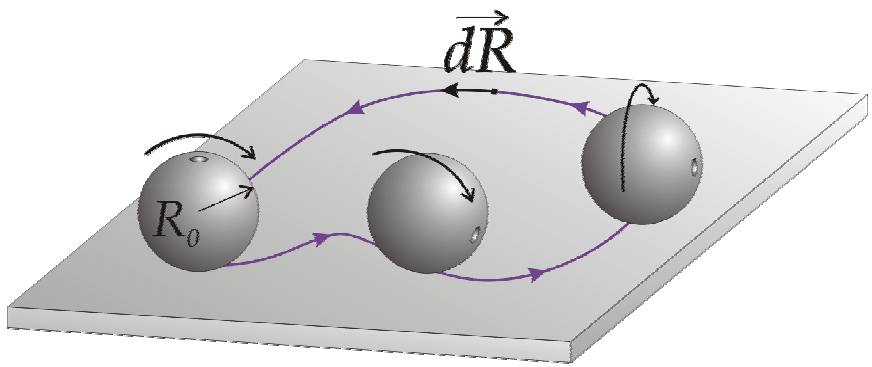
Coish *et al*., Phys. Stat. Sol. 06

$$U[d\vec{r}] = \exp\left(\frac{i}{2}\,\vec{\sigma}\cdot\vec{B}_{\rm so}\,dt\right)$$

$$= \exp\left(\frac{i}{2}\,\vec{\sigma}\cdot\lambda_{\rm so}^{-1}\cdot d\vec{r}\right)$$

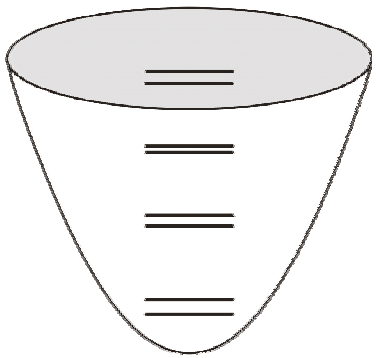
$$d\vec{r} = \frac{\vec{p}}{m^*}dt \quad , \quad \lambda_{\rm so}^{-1} = m^*\begin{pmatrix} -\beta & \alpha \\ -\alpha & \beta \end{pmatrix}$$





moving electron + spin orbit interaction ⇔ ball rolling on surface

Electron with spin in quantum dot



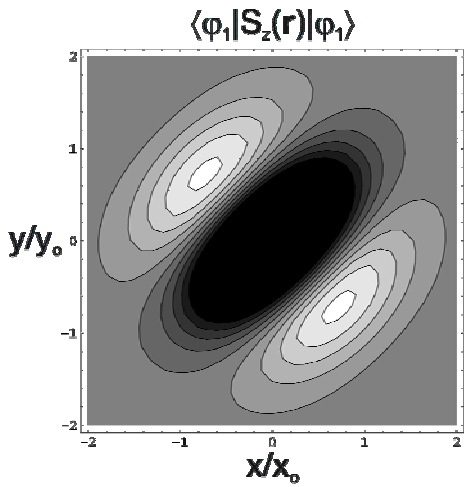
$$H(t) = \frac{p^2}{2m} + V(\vec{r}) + H_{\mathrm{SO}}$$

electric field ⇒ displacement

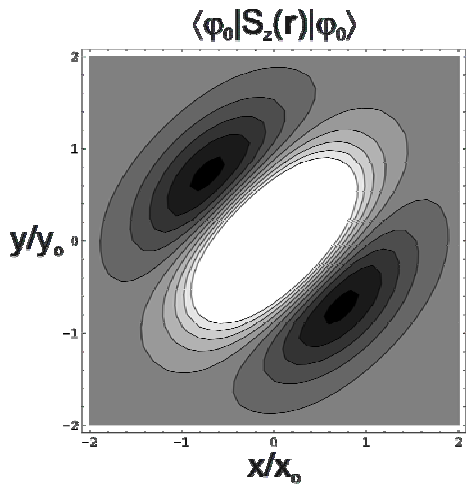
$$V(\vec{r}) = \frac{1}{2} m\omega_0^2 r^2 \;\Rightarrow\; V(\vec{r}) + e\vec{E}(t)\vec{r} = V\left(\vec{r} + \overbrace{\frac{e\vec{E}(t)}{m\omega_0^2}}^{-\Delta\vec{r}(t)}\right)$$

Spin-orbit coupling

- Eigenstates are *spin-textures*
- For B=0 the states are two-fold degenerate (Kramer's theorem)
- The lowest doublet is labeled by pseudospin τ



τ = "↓"



τ = "↑"

Slow variation of $\vec{E}(t)$ ***⇒ Adiabatic evolution of lowest doublet***

Spin relaxation and decoherence

Noisy electric field leads to spin diffusion

For weak/slow electric fields → effective Hamiltonian

$$H_{\rm eff} = \underbrace{-\frac{1}{2}\vec{B}_{\rm eff}\vec{\tau} + \ldots}_{} + \underbrace{\frac{x_0^2 e^2}{2}\left(\dot{E}_\mu E_\nu - E_\mu \dot{E}_\nu\right)\vec{C}_{\mu\nu}\vec{\tau}}_{}$$

vanish at zero magnetic field

geometric contribution
requires two orthog. fluctuating fields

Dominant contributions to electric noise:

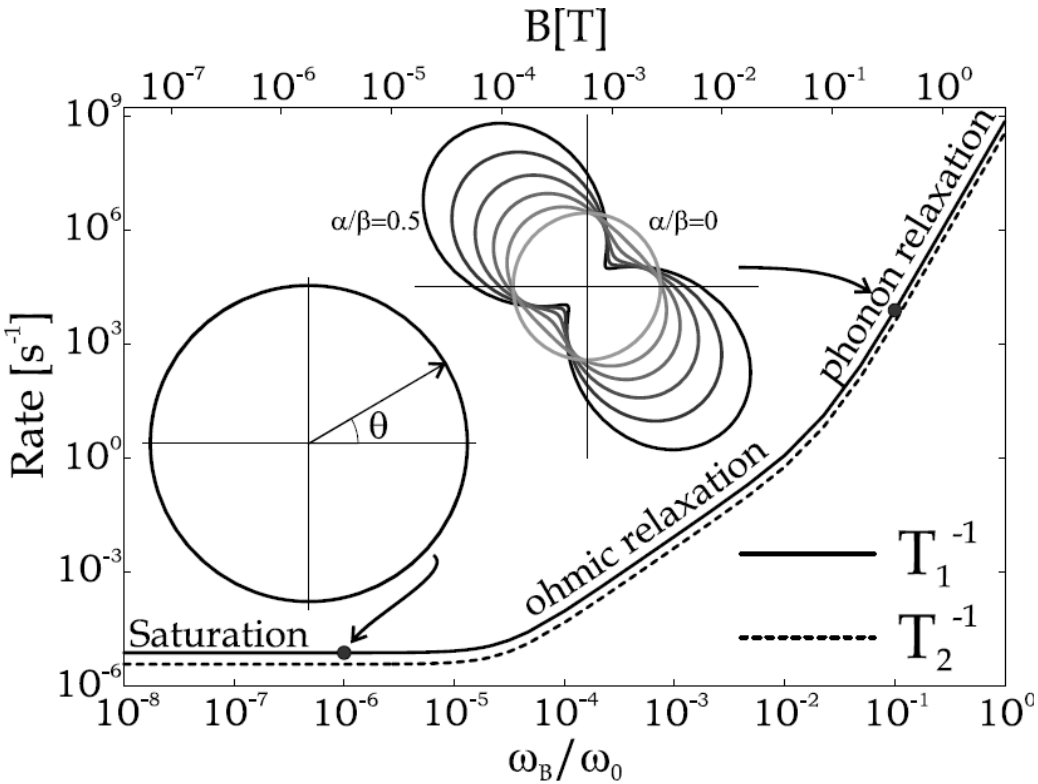
- Piezo-electric longitudinal phonons: $\rho_{\rm ph}(\omega) \propto \omega^3$
- Ohmic fluctuations from the leads: $\rho_{\Omega}(\omega) \propto \omega$

(dominant at low energies)

Spin relaxation and decoherence rates

GaAs/AlGaAs
lateral quantum dot

T = 0.1K
dot size = 50 nm
l_{SO}=3 μm
α/β=1/4



Saturation at low B

$$\frac{1}{T_1} \propto S_{\dot{E}_x E_y - E_x \dot{E}_y}(0)$$

Decay due to random geometric phase
due to 2 fluctuating field components

$$S_{\dot{E}_x E_y - E_x \dot{E}_y}(\omega) \approx 2 \int d\tilde{\omega}\, \tilde{\omega}^2\, S_{E_x}\left(\frac{\omega+\tilde{\omega}}{2}\right) S_{E_y}\left(\frac{\omega-\tilde{\omega}}{2}\right)$$

Conclusions

- Progress with solid-state qubits
  - Josephson junction qubits (exps. 1999+)
  - spins in quantum dots (exps. 2005+)
- Crucial: understanding and control of decoherence
  - sources and properties of noise,
  - coupling mechanisms, decay laws
- Present activities:
  - coupling of qubits, coupling of qubit and resonator, …
  - pumping, read-out, …
- Spin-offs:
  - combine nano-electronics, low-T, and high-frequency technology
  - qubits as spectrum analyzer of noise
  - quantum optics effects: single-qubit lasing, cooling, …

Entanglement

S. C. , Xun Jia, A. Kopp

I. Gruzberg, A. Subramanium

- Entanglement as a characterization of a ground state wave function.
- A measure of entanglement: Von Neumann entropy
- Entanglement and quantum phase transition.
- 3D Anderson localization.
- Integer quantum Hall plateau transition.

"When two systems, of which we know the states by their respective representatives, enter into temporary physical interaction due to known forces between them, and when after a time of mutual influence the systems separate again, then they can no longer be described in the same way as before, viz. by endowing each of them with a representative of its own. I would not call that one but rather the characteristic trait of quantum mechanics, the one that enforces its entire departure from classical lines of thought. By the interaction the two representatives [the quantum states] have become entangled."
- Schrödinger (Cambridge Philosophical Society, 1935)



Von Neumann entropy as a measure of entanglement

Consider bipartite system and define

$$\rho_{AB} = |\psi_{AB}\rangle\langle\psi_{AB}|$$

Then the reduced density matrix

$$\rho_A = \mathrm{Tr}_B\ \rho_{AB}$$

is in general a mixture iff the parts are entangled, otherwise it is pure. Then the Von Neumann entropy of *A* is a measure of entanglement

$$S = -\mathrm{Tr}\rho_A \ln \rho_A$$

From the Schmidt decomposition theorem it follows that

$$S = -\mathrm{Tr}\rho_A \ln \rho_A = -\mathrm{Tr}\rho_B \ln \rho_B$$

Schmidt : $|\psi\rangle = \sum_i \sqrt{\mu_i}|i_A\rangle|i_B\rangle,\ \mu_i \geq 0,\ \sum_i \mu_i = 1$

Not true : $|\psi\rangle = \sum_i \sqrt{\mu_i}|i_A\rangle|i_B\rangle|i_C\rangle$

Quantum critical points as highly entangled states

$$S_{\rm thermal} = -\left(\beta\frac{\partial}{\partial\beta} - 1\right)\ln {\rm Tr}\ e^{-\beta H}$$

$$S_{\rm vNeumann} = -\left(n\frac{\partial}{\partial n} - 1\right)\ln {\rm Tr}\ \rho_A^n|_{n\to 1+}$$

$${\rm Tr}\ \rho_A^n = \sum_i \mu_i^n < \sum_i \mu_i = 1,\ \ \mu_i : [0,1)$$

The l.h.s. is absolutely and uniformly convergent and therefore analytic for all ${\rm Re}\ n > 1$. The non-analyticity must be in ρ_A

- Conjecture: the Von Neumann entropy is non-analytic at a quantum phase transitions, in particular at a quantum critical point. It is otherwise analytic.

- There are cases where the ground state energy is analytic but the Von Neumann entropy is non-analytic at the quantum phase transition . This is not the traditional picture of a quantum phase transition, for example, Ising model in a transverse field.

Quantum phase transition and quantum critical point

One of the simplest examples is

$$\mathcal{H} = -J\lambda \sum_i S_i^z S_{i+1}^z - J \sum_i S_i^x$$

1. When $\lambda \to 0$ the ground state is the direct product state of qubits---not entangled

$$|0\rangle = \prod_i \left(\frac{|\uparrow\rangle + |\downarrow\rangle}{\sqrt{2}} \right)_i$$

Note that it is not the same as the state

$$|0\rangle_S = \frac{1}{\sqrt{2}} \left(\prod_i |\uparrow\rangle_i + \prod_i |\downarrow\rangle_i \right)$$

2. When $\lambda \to \infty$ the ground state is the unentangled direct product

$$|0\rangle = \prod_i |\uparrow\rangle_i$$

What happens when λ increases from 0 ?

More and more qubits get entangled, but the symmetry remains unbroken until $\lambda = 1$

At $\lambda = 1$, there is a quantum phase transition to a broken symmetry state-- $\lambda = 1$ is a quantum critical point

The second derivative of the ground state energy density is logarithmically singular $\lambda = 1$

$$e_0 = -\frac{2J}{\pi}(1+\lambda)E\left(\frac{2\sqrt{\lambda}}{1+\lambda}\right)$$

It is also easy to compute the single site Von Neumann entropy, where the subsystem A is the site and B is the rest.

$$\rho_A = \frac{1}{2}\begin{pmatrix} 1+\langle\sigma_i^z\rangle & \langle\sigma_i^x\rangle \\ \langle\sigma_i^x\rangle & 1-\langle\sigma_i^z\rangle \end{pmatrix}$$

$$\lim_{\lambda\to 1+}\frac{\partial S}{\partial\lambda} = -\frac{\pi}{2^{19/4}}\ln\left(\frac{\pi+2}{\pi-2}\right)(\lambda-1)^{-3/4}$$

$$\lim_{\lambda\to 1-}\frac{\partial S}{\partial\lambda} = -\frac{1}{2\pi}\ln\left(\frac{\pi+2}{\pi-2}\right)\ln|\lambda-1|$$

Anderson localization

$$H = \sum_n \varepsilon_n c_n^\dagger c_n - t \sum_{\langle m,n \rangle} \left(c_n^\dagger c_m + h.c. \right)$$

$$-W/2 \leq \varepsilon_n \leq W/2$$

All states are localized for dimensions $D \leq 2$but there is a metal-insulator transition for $D = 3$ for a critical W/t

A theorem due to Thouless

Impurity averaged ground state energy is analytic.

$$\overline{G}_{mn} = \int G_{mn}(E, \{\varepsilon_n\}) d\mu\{\varepsilon_n\}$$

The density of states must be analytic in the real energy range

$$-\frac{W}{2} + Zt \leq E \leq \frac{W}{2} - Zt$$

$$\rho_n = z_n(|1\rangle\langle 1|)_n + (1-z_n)(|0\rangle\langle 0|)_n,\ z_n = |\psi_n^\alpha|^2$$

$$S_{Vn} = -z_n \ln z_n - (1-z_n)\ln(1-z_n)$$

$$\overline{S_V} = \frac{1}{MN}\overline{\sum_{\alpha,n} S_{Vn}^\alpha}$$

From multifractal analysis

$$L^d \overline{S_V} = \mathcal{C} + (\ln L) L^{-1/\nu} f(L^{1/\nu} w)$$

$$\overline{S_V}(E=0,w) = k(w)\ln L + C(w)$$

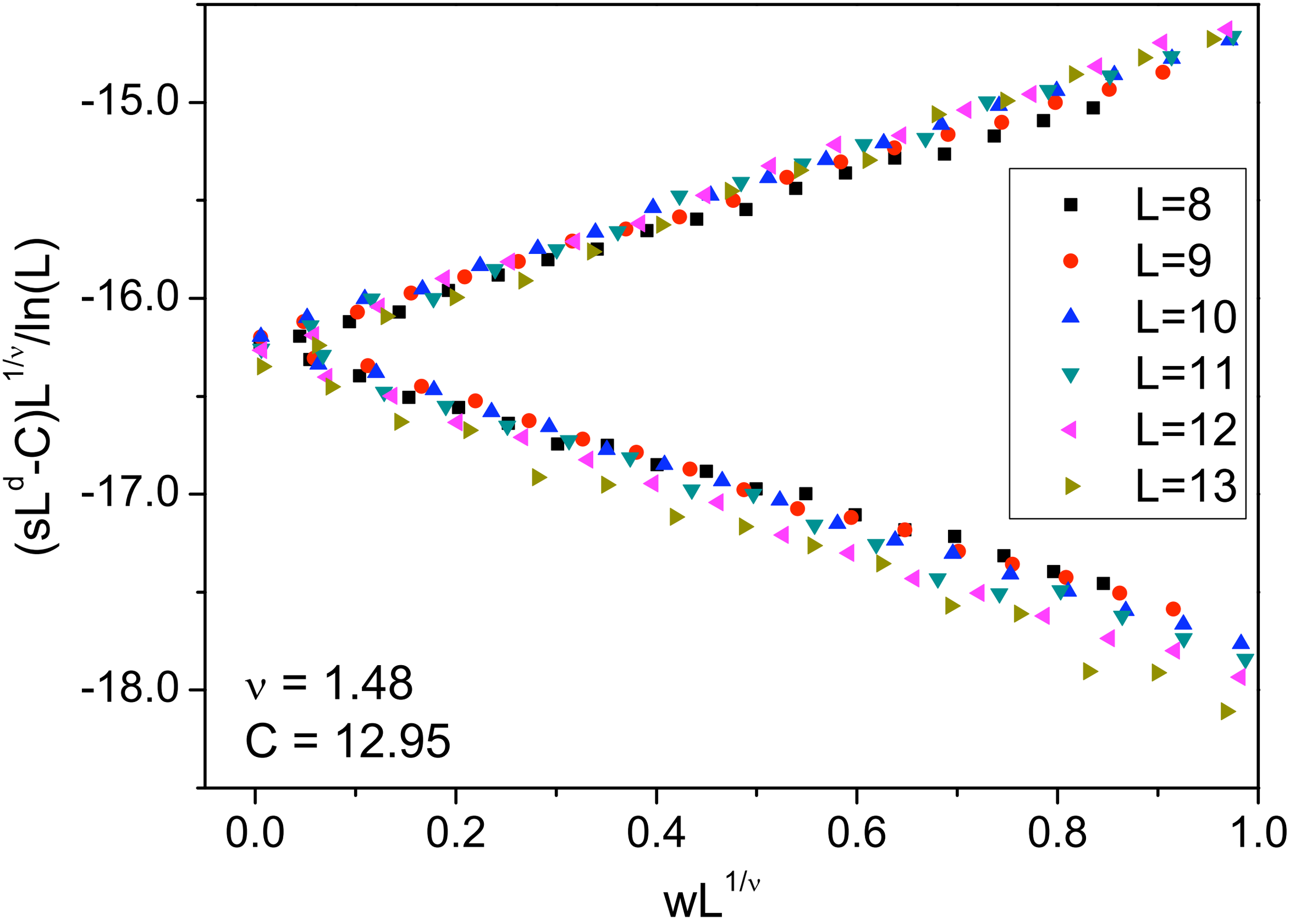
$$k(w) \quad \rightarrow \quad d,\ \text{metallic}$$

$$\rightarrow \quad 0,\ \text{insulating}$$

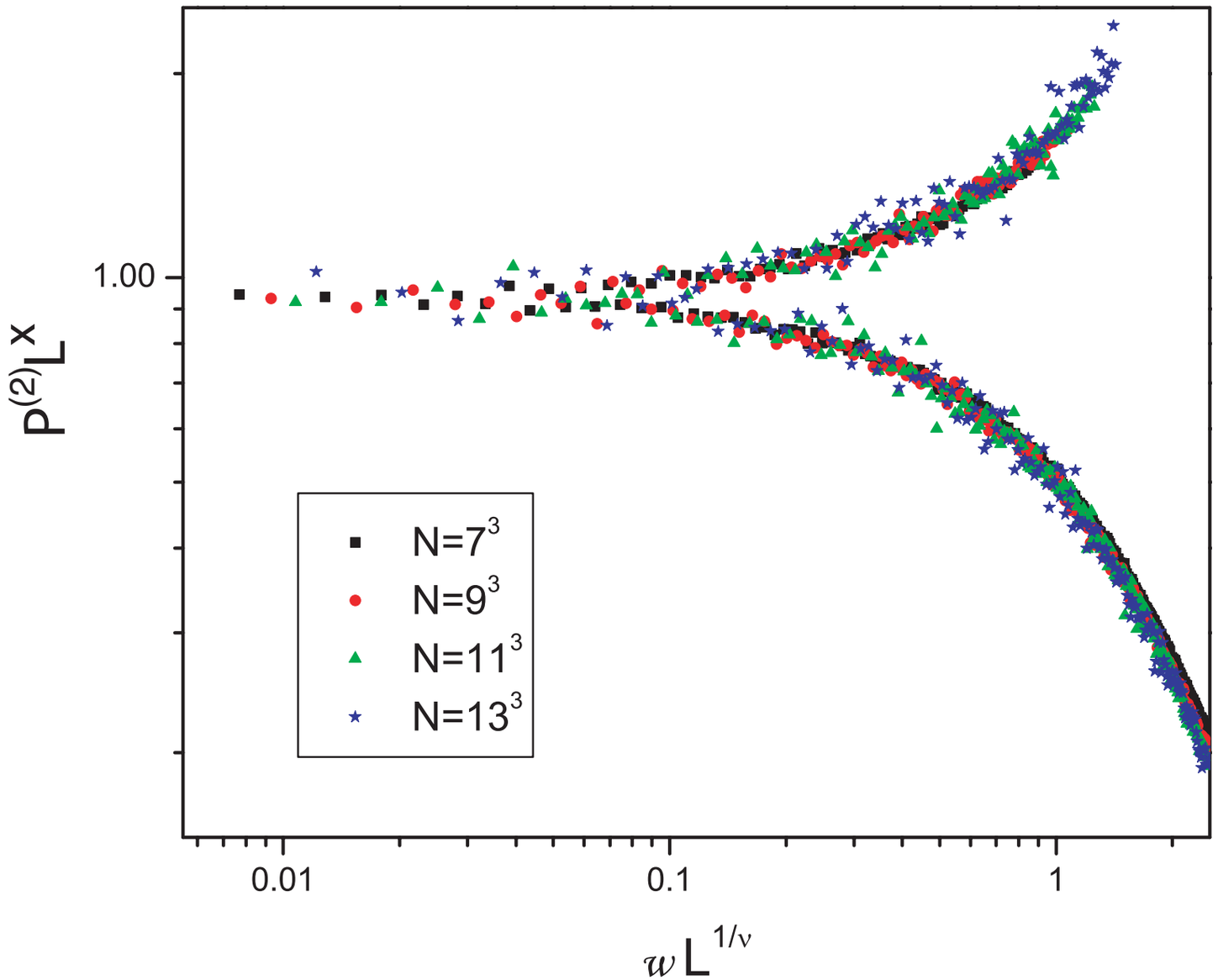
$$\rightarrow \quad \alpha_1 = 2,\ w = 0$$

$$L = N^{1/3}$$

$$w = |W - W_c|/W_c$$


-15.0
-16.0
-17.0
-18.0
(sL^d-C)L^1/ν/ln(L)
ν = 1.48
C = 12.95
L=8
L=9
L=10
L=11
L=12
L=13
0.0
0.2
0.4
0.6
0.8
1.0
wL^1/ν

$$W_c = 16.5$$

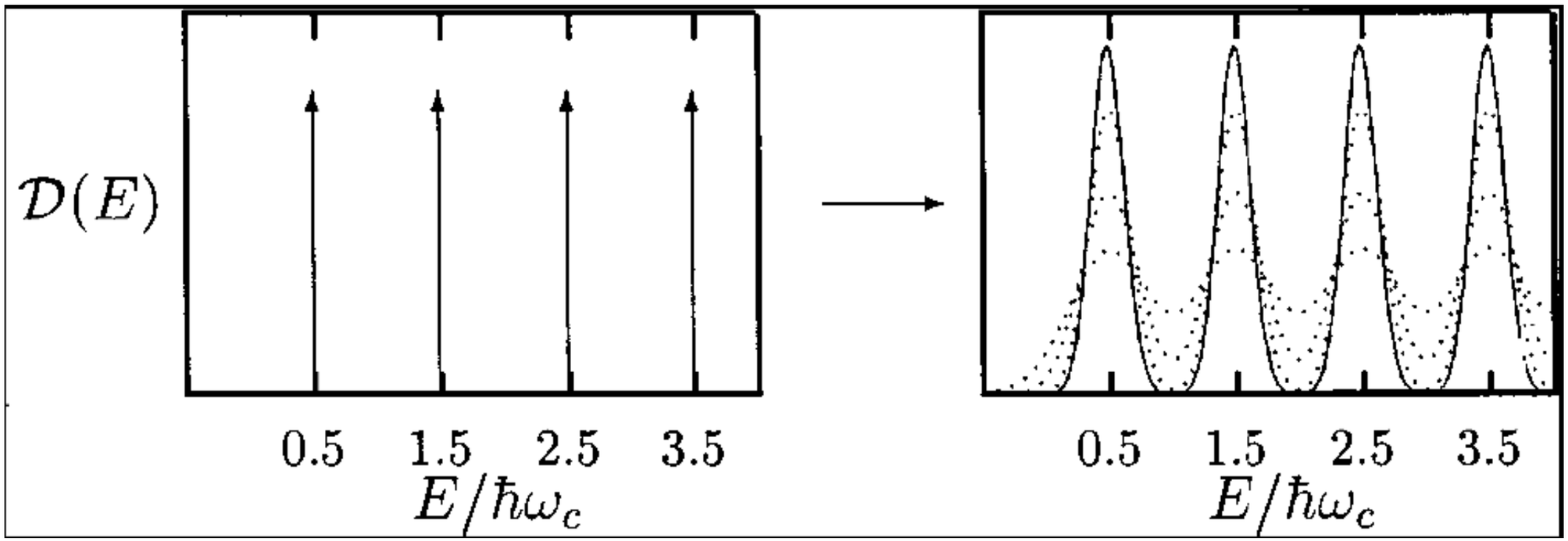


$$S_L = -\mathrm{Tr}\rho_\mathrm{A}(\rho_\mathrm{A} - 1) = 1 - \mathrm{Tr}\rho_\mathrm{A}^2$$

$$P^{(2)} = \frac{1}{\mathcal{A}} \sum_{i,\alpha} |\psi_i^\alpha|^4 = 1 - S_L/2 \qquad \text{(participation ratio)}$$

$$P^{(2)} = L^{-x} g_\pm(L^{1/\nu} w) \qquad x = 1.4$$

Integer Quantum Hall effect



$$H = \sum_{n,k} |n,k\rangle\langle n,k| \left(n + \frac{1}{2}\right)\omega_c + \sum_{n,k}\sum_{n',k'} |n,k\rangle\langle n,k|V|n',k'\rangle\langle n',k'|$$

Projected to the lowest Landau level: $\langle 0,k|V|0,k'\rangle$

$$L = L_x = L_y = \sqrt{2\pi} M l_B \qquad l_B = (\hbar/eB)^{1/2}$$

$$k = 2m\pi/L \qquad m = 0, 1, \ldots, N_k - 1 \qquad N_k = BL^2/\phi_0 = 2M^2$$

$$|\psi\rangle = \sum_k \alpha_k |0,k\rangle$$

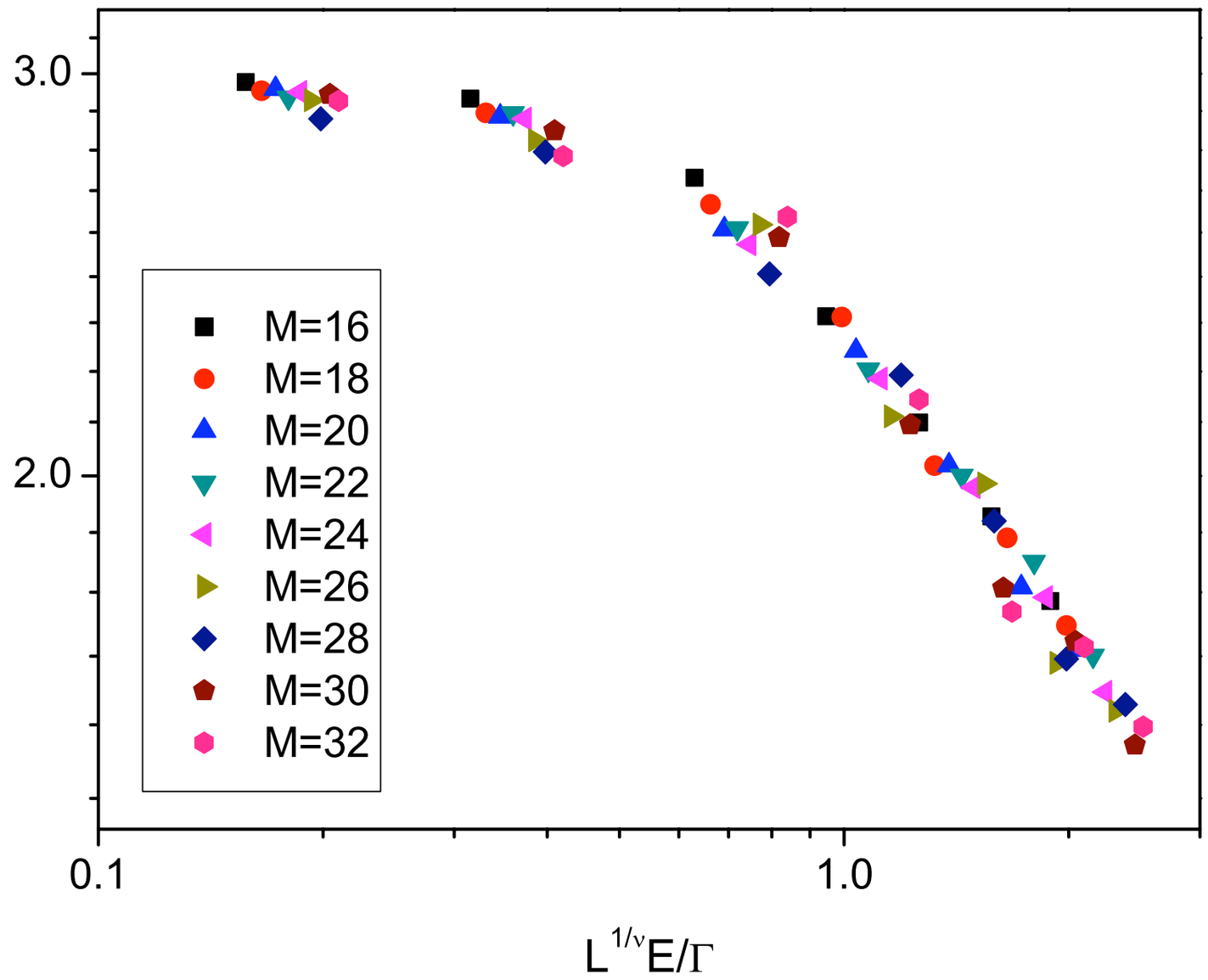
$$\psi(x,y) = \langle x,y|\psi\rangle = \sum_k \alpha_k \psi_{0,k}(x,y)$$

Plateau transition

$$A : (\pi/2)(l_B)^2$$

$$z = \int_{(x,y)\in A} |\psi(x,y)|^2 dxdy$$

$$S = -z \ln z - (1-z)\ln(1-z)$$



E is measured in units of

$$\Gamma = 2V_0/\sqrt{2\pi} l_B$$

$$\overline{V(r)V(r')} = V_0^2 \delta(r-r')$$

$$\nu \approx 7/3$$

Conclusions

- Entanglement entropy seems to correctly capture quantum phase transitions and may provide additional insight.

- Entanglement entropy may be an effective way of detecting hidden orders in interesting many body ground state.

50 Years of Condensed Matter Theory

Symposium in honor of Vinay Ambegaokar
Ithaca, June 16, 2007

„Supervision by benevolent neglect…"

„What, if anything, have you done lately?"

„I have turned the pages of your draft.
Quite frankly, I found it to be unreadable…"

„He emphasized at least 25 times that his results were exact, but when asked, had to admit that his model was based on an approximation!"

„Follow the phenomena!"

Mesoscopic to Universal Crossover of Transmission Phase of Multi-Level Quantum Dots

Arnold Sommerfeld Center for Theoretical Physics

CeNS Center for NanoScience

Jan von Delft

nsm nanosystems initiative munich

SFB 631



Christoph Karrasch



Theresa Hecht



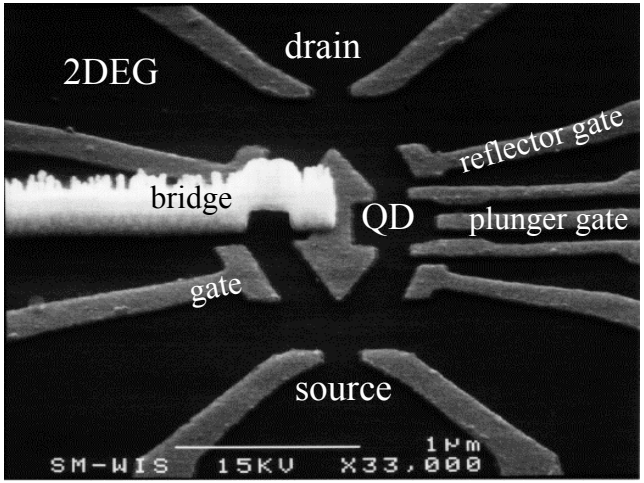
Andreas Weichselbaum



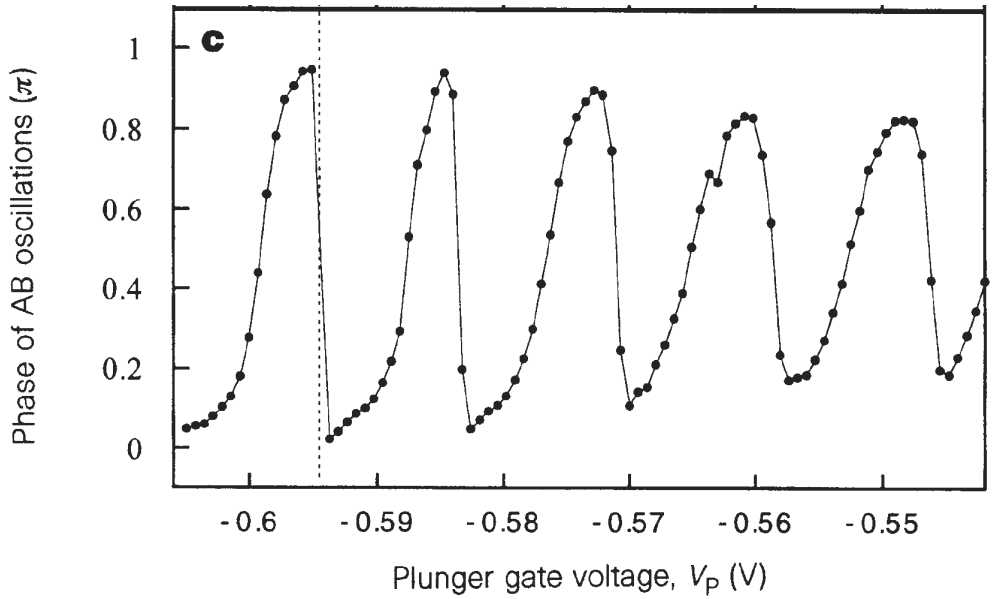
Yuval Oreg



Volker Meden



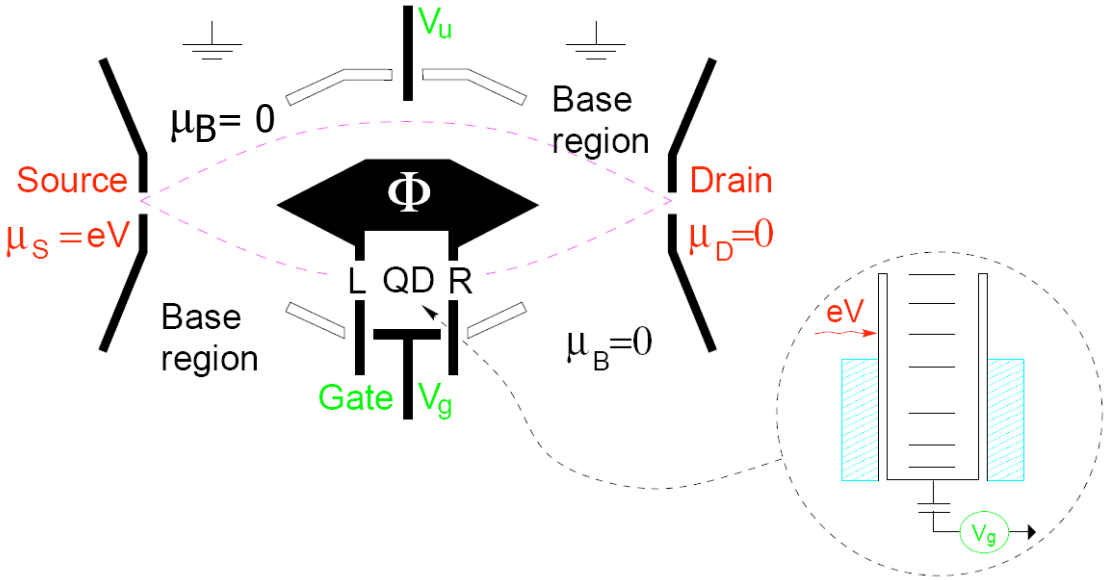
Heiblum group: Schuster et al, Nature, 1997

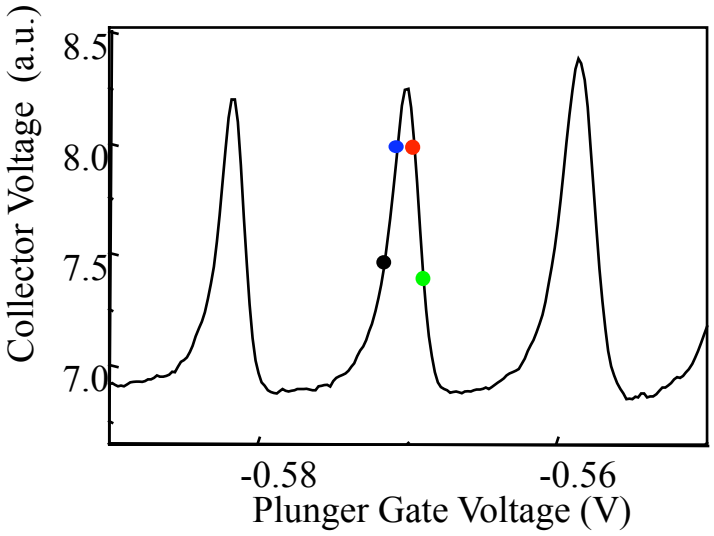


Aharonov-Bohm interferometry: experimental setup

Yacoby et al., PRL **74**, 4047 (1995)

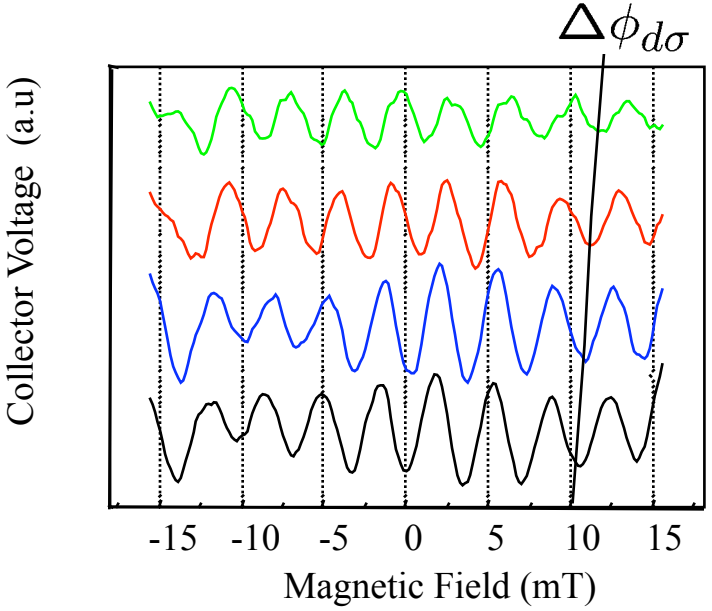
Schuster et al. Nature **385**, 417 (1997)



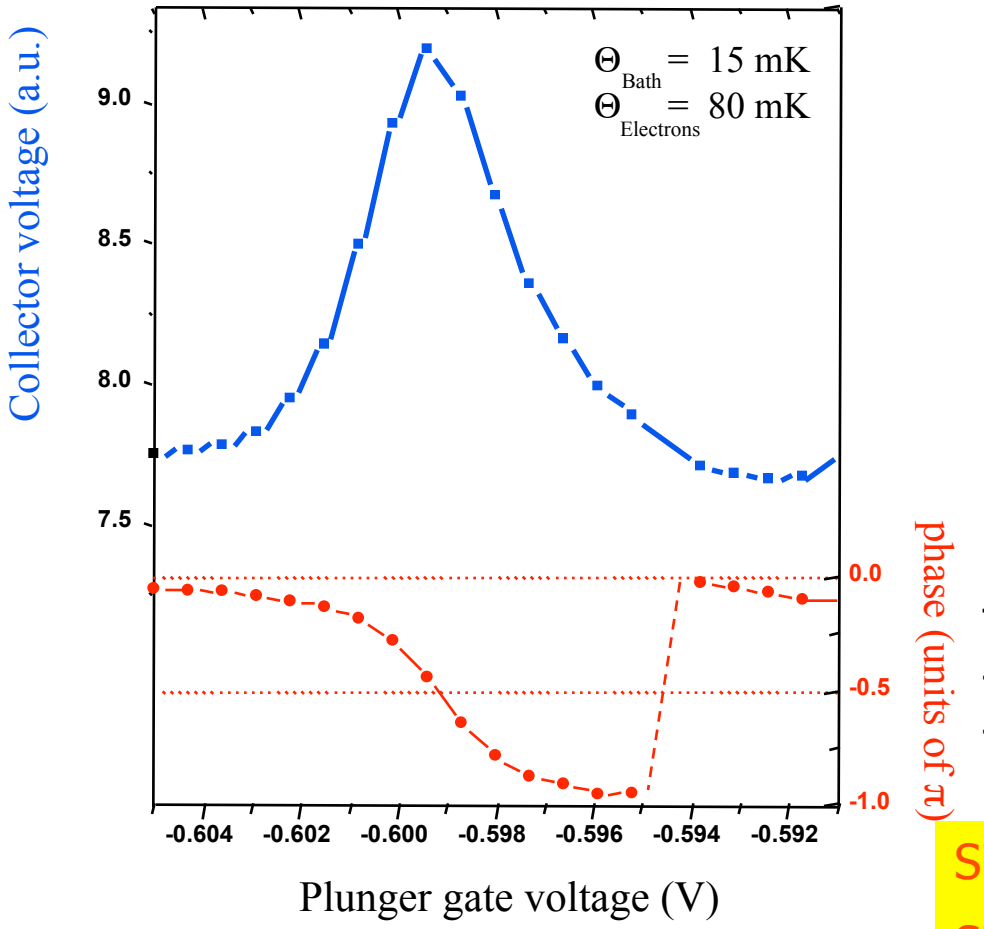


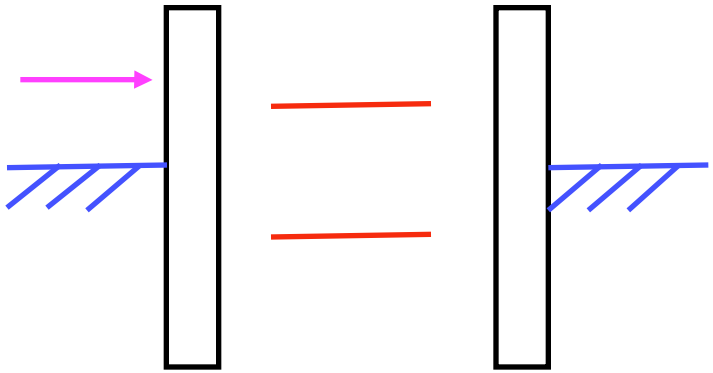
$$|t_{SD}|^2 = \sum_\sigma \left| t_{u\sigma}\, e^{i2\pi\Phi/\Phi_0} + t_{d\sigma} \right|^2$$

$$= \sum_\sigma \Big[|t_{u\sigma}|^2 + |t_{d\sigma}|^2 + 2|t_{u\sigma}||t_{d\sigma}| \cos(2\pi\Phi/\Phi_0 + \phi_{u\sigma} - \phi_{d\sigma}) \Big]$$



Expected Phase Evolution for Resonant Tunnelling





$$t_{d\sigma} \propto \frac{t_i^L t_i^R}{(\varepsilon - \varepsilon_i) + i\Gamma_i} + \frac{t_j^L t_j^R}{(\varepsilon - \varepsilon_j) + i\Gamma_j}$$

Two terms can cancel completely. Then transmission amplitude = 0, transmission phase jumps by π !

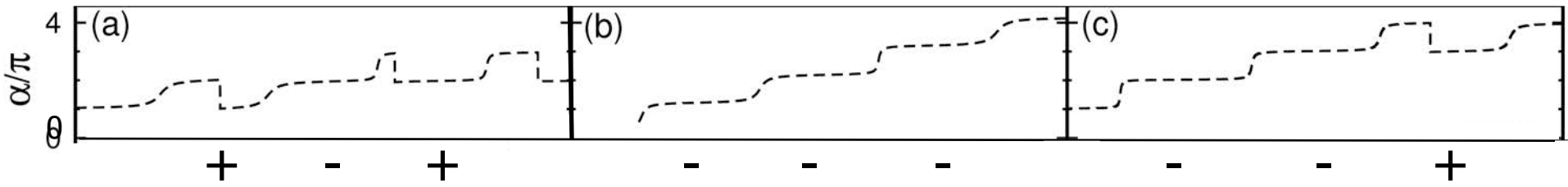
Abrupt phase lapse between the peaks, with width $< G_e$, $< k_B T$

Simple explanation for phase lapse:

Sign change of transmission amplitude, due to „destructive interference" between two partially overlapping levels:

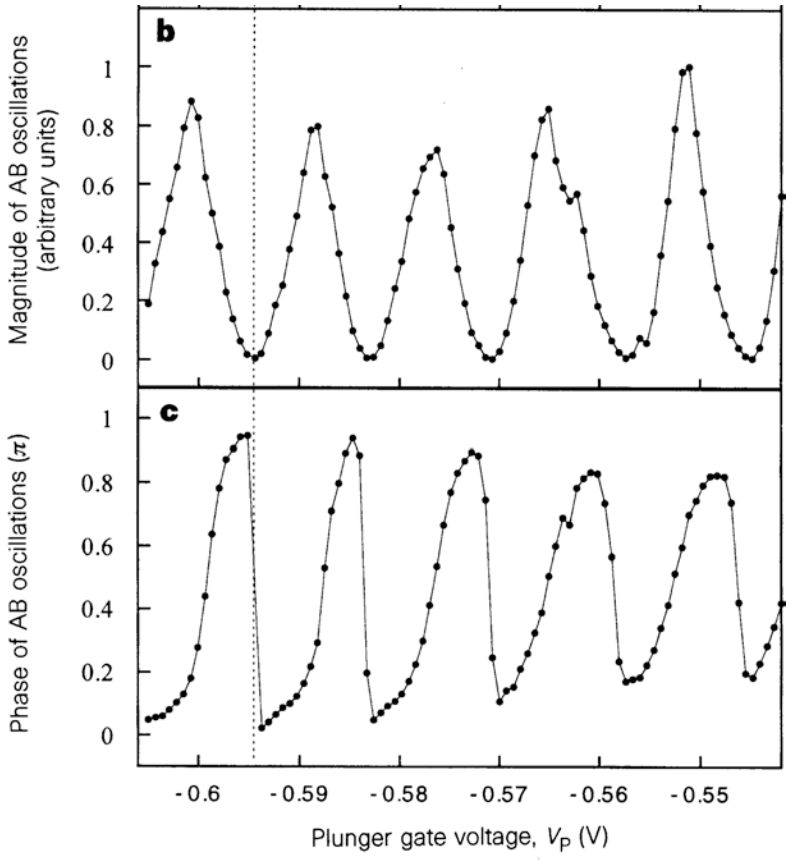
Moty Heiblum's phase lapse puzzle...

Prediction of model of noninteracting electrons:



signs: $s_j = \mathrm{sgn}(t_j^L t_j^R t_{j+1}^L t_{j+1}^R) = \pm$

Schuster et al. Nature 1997



Results for many-electron quantum dots:

- Phase lapse occurs in <u>every</u> conductance valley!
- "Universal phase lapse behavior"
- Sample-specific details not matter!??
- Triangular conductance peak shapes!

Some of theory papers addressing phase lapse puzzle...

Schuster et al: > 230 citations, > 20 theory PRLs

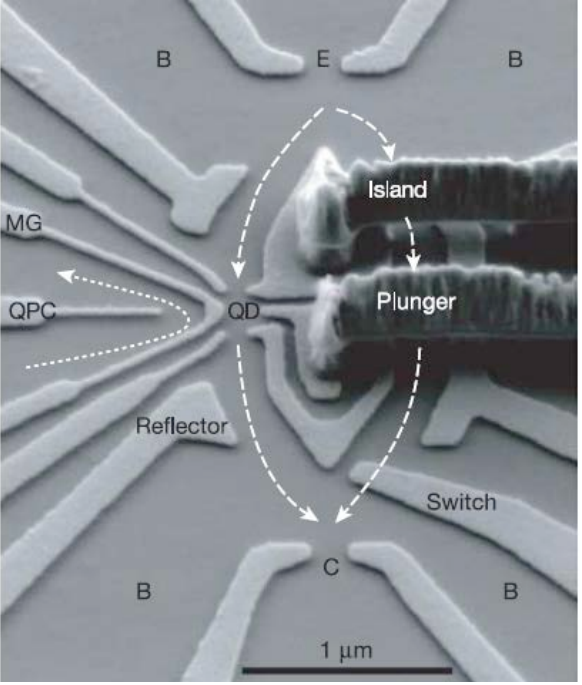
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9. **P.G. Silvestrov and Y. Imry, Phys. Rev. Lett. 85, 2565 (2000) [one wide level, many narrow levels]**
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19. **P.G. Silvestrov and Y. Imry, Phys. Rev. B. 65, 035309 (2002) [one wide level, many narrow levels]**
20. Y. Gefen, in Quantum Interferometry with Electrons: Outstanding Challenges Lerner I V et al. (eds) (Kluwer, Dordrecht) p. 13 (2002)
21. T-S. Kim, and S. Hershfield, Phys. Rev. B 67 235330 (2003)
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26. M. Sindel, A. Silva, Y. Oreg, and J. von Delft, Phys. Rev. B 72, 125316 (2005)
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Moty Heiblum: "No single acceptable explanation !"

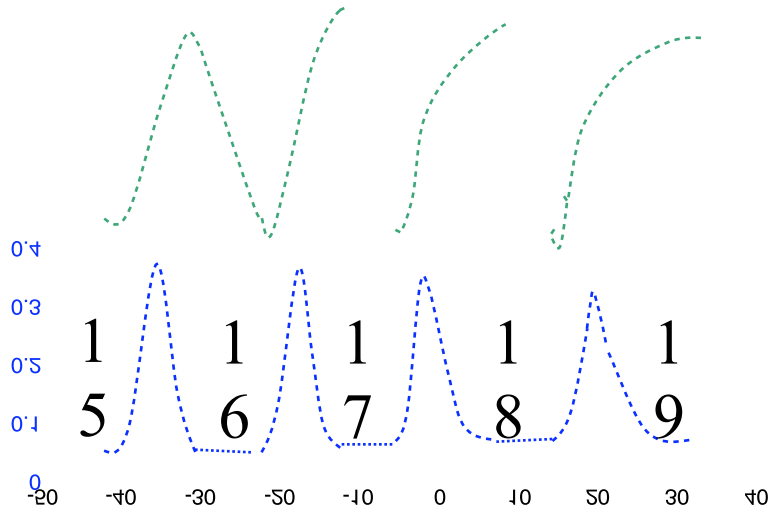
28. S. Kim and H.-W. Lee, Phys. Rev. B 73, 205319 (2006)
29. **D. I. Golosov and Y. Gefen, Phys. Rev. B 74, 205316 (2006) [non-symmetric couplings to leads]**
30. **C. Karrasch, T. Hecht, Y. Oreg, J. von Delft, and V. Meden, Phys. Rev. Lett. 98, 186802 (2007)** [wide+narrow levels, Fano effect]
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32. **C. Karrasch, T. Hecht, A. Weichselbaum, J. von Delft, Y. Oreg, V. Meden, cond-mat/0612490, to appear in New J. Phys.**
33. **P. G. Silvestrov and Y. Imry, cond-mat/0609355, Phys. Rev. B 2007, to appear in New J. Phys.**
34. **Y. Oreg, to appear in New J. Phys. [wide level, no interactions]**

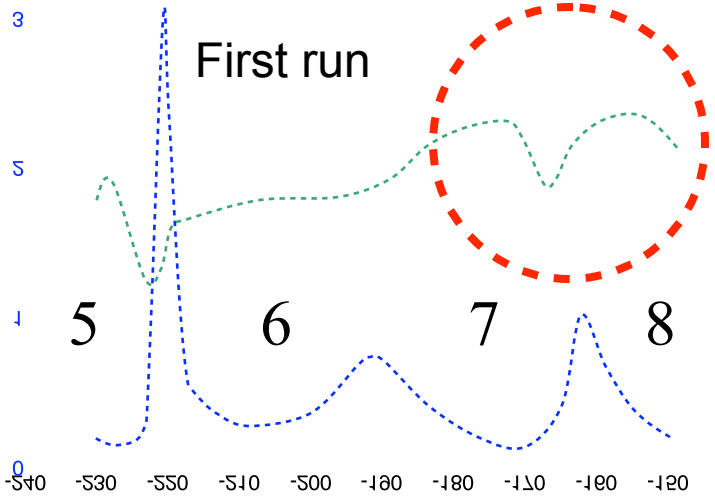
Moty Heiblum: "theoretical picture seems to be converging..."

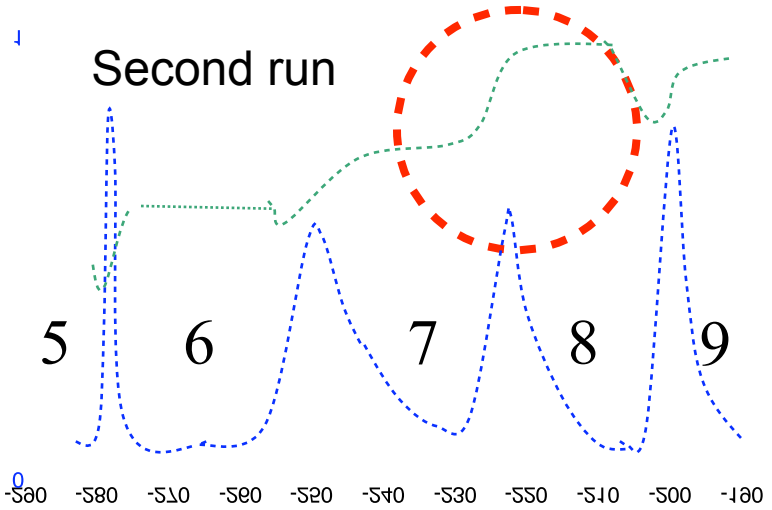
Few-electron AB interferometry



M. Avinun-Kalish,
M. Heiblum,
O. Zarchin,
D. Mahalu,
V. Umansky,
Nature, 2005







Crossover from ‘universal’ to ‘mesoscopic’ behavior

Summary of experimental findings:

(1) For occupation 1 to about 10, the phase evolution is highly sensitive to the dot's configuration and occupation.

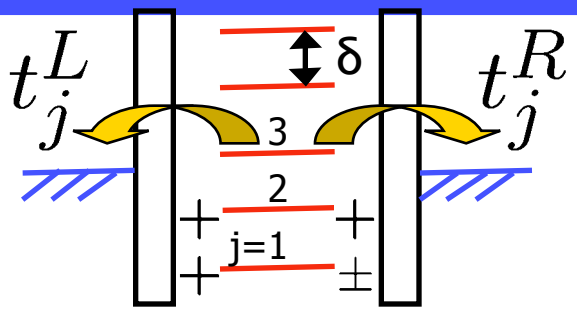
(2) For occupation higher than about 14, phase evolution is universal-like, with phase ranging only between 0 and π, and is independent of the dot's configuration and occupation.

Speculation on possible mechanism:

„An outstanding feature of larger quantum dots is the smaller level spacing, which might lead to levels overlapping. This will favour models that invoke transmission mediated through more than one quantum state." Avinun-Kalish et al., Nature 2005

Multi-level Dot Model (spinless electrons)

$$H_{\text{lead}} = \sum_{k}^{L,R} \sum_{\mu} \varepsilon_{k\mu} c^{\dagger}_{k\mu} c_{k\mu}$$



$$H_{\text{dot}} = \sum_{ij} \underbrace{\left(\varepsilon_j - V_g\right) \delta_{ij}}_{h^0_{ij}} d^{\dagger}_i d_j + \tfrac{1}{2} U \sum_{i \neq j} (\hat{n}_i - \tfrac{1}{2})(\hat{n}_j - \tfrac{1}{2})$$

$$\delta = \varepsilon_{j+1} - \varepsilon_j$$

$$H_{\text{lead-dot}} = \sum_{k} \left[c^{\dagger}_{kL} \sum_j t^L_j d_j + c^{\dagger}_{kR} \sum_j t^R_j d_j \right]$$

$$\delta \gg \Gamma$$
$$\delta \ll \Gamma$$

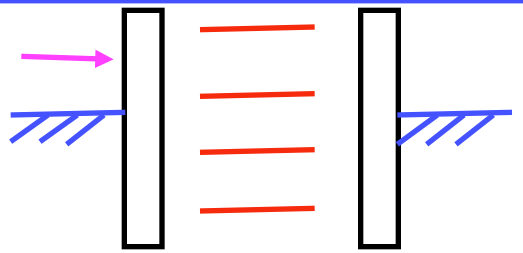
Partial widths : $\Gamma^{\mu}_j = \pi \rho |t^{\mu}_j|^2,$ Average : $\Gamma = \frac{1}{N} \sum_{j\mu} \Gamma^{\mu}_j$

Signs of t^{μ}_j : $s_j = \text{sgn}(t^L_j t^R_j t^L_{j+1} t^R_{j+1}) = \pm$

Collective notation : $\gamma = \{\Gamma^L_1, \Gamma^R_1, \Gamma^L_2, \ldots\},$ $\sigma = \{s_1, s_2, \ldots\}$

Transmission Amplitude

derived from results of: Bruder, Fazio, Schoeller PRL, **76**, 114 (1996)



$$T_{LR} = 2\pi\rho \sum_{ij} t_i^L \, \overline{G_{ij}^{0R}} \, t_j^R = |T| e^{i\alpha}$$

$$\overline{G_{ij}^{0R}} = \int d\varepsilon \underbrace{[-\partial_\varepsilon f(\varepsilon)]}_{\delta(\varepsilon) \text{ for } T=0.} G_{ij}^{0R}(\varepsilon)$$

Assumptions:
1. phase-coherent transport
2. only one mode carries current
3. no multiple traversals of ring
4. dot stays in equilibrium

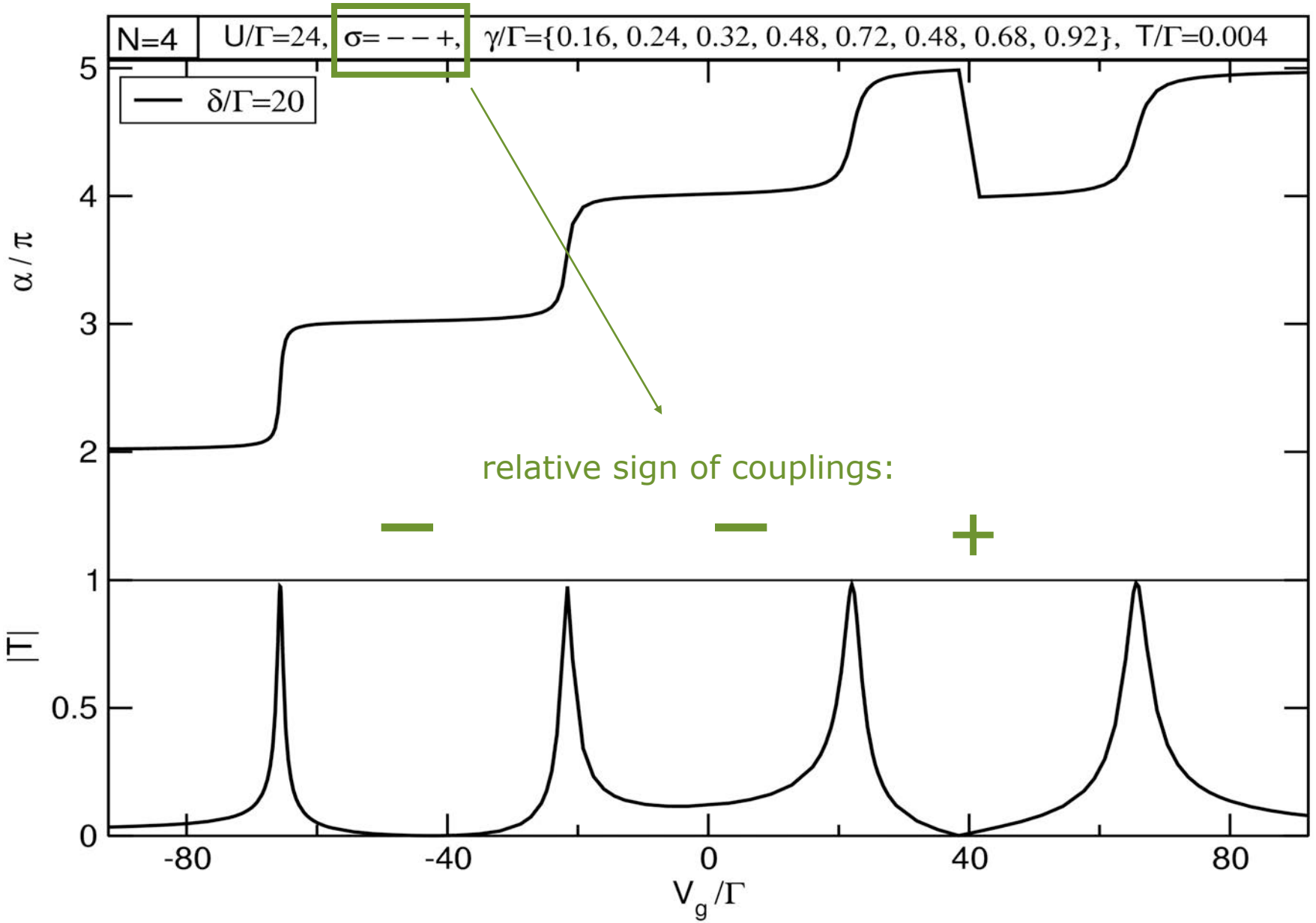
Numerical Renormalization Group (NRG)
[iterative diagonalization of Hamiltonian]

$$\left.\begin{matrix} \varepsilon_j, t_j^\mu \\ U, V_g \end{matrix}\right\} \rightarrow \boxed{\text{NRG}} \rightarrow \mathcal{G}_{ij}^R(\varepsilon, T)\Big|_{V_g}$$

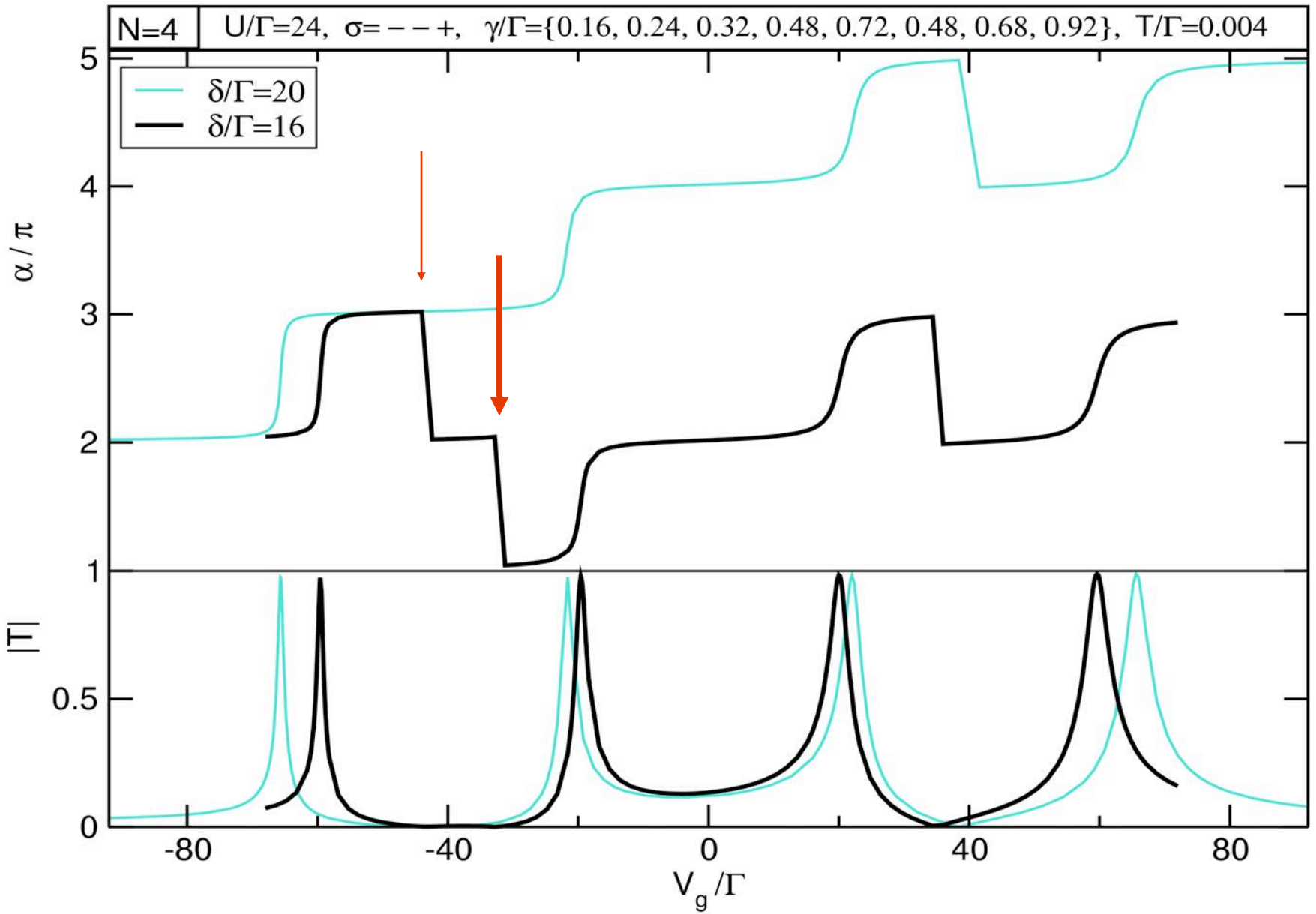
Functional Renormalization Group (fRG)
[RG-improved perturbatrion theory in U]

$$\left.\begin{matrix} \varepsilon_j, t_j^\mu \\ U, V_g \end{matrix}\right\} \rightarrow \boxed{\text{fRG}} \rightarrow \mathcal{G}_{ij}^R(\varepsilon=0, T=0)\Big|_{V_g}$$

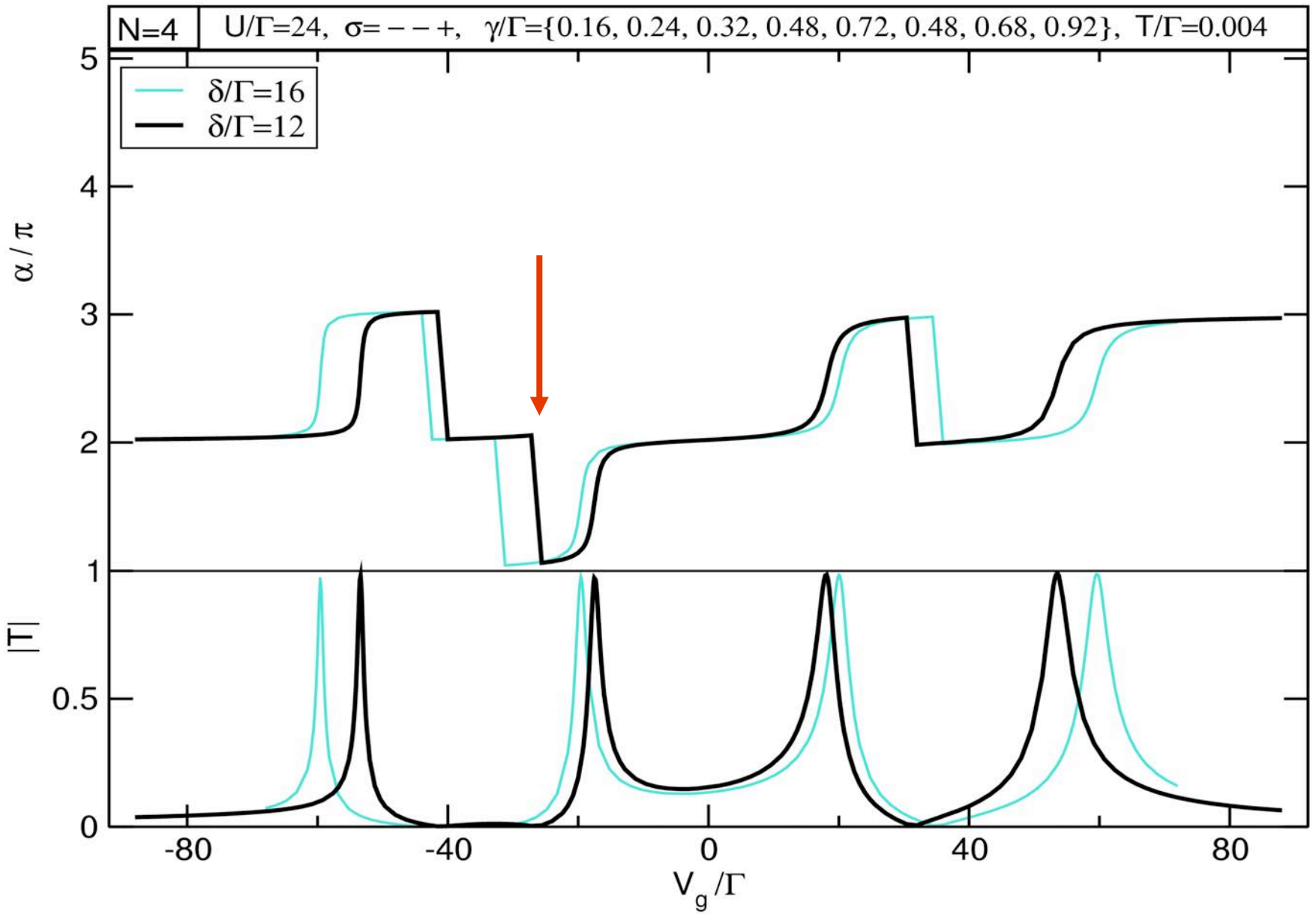
NRG: δ/Γ = 20 (mesoscopic)



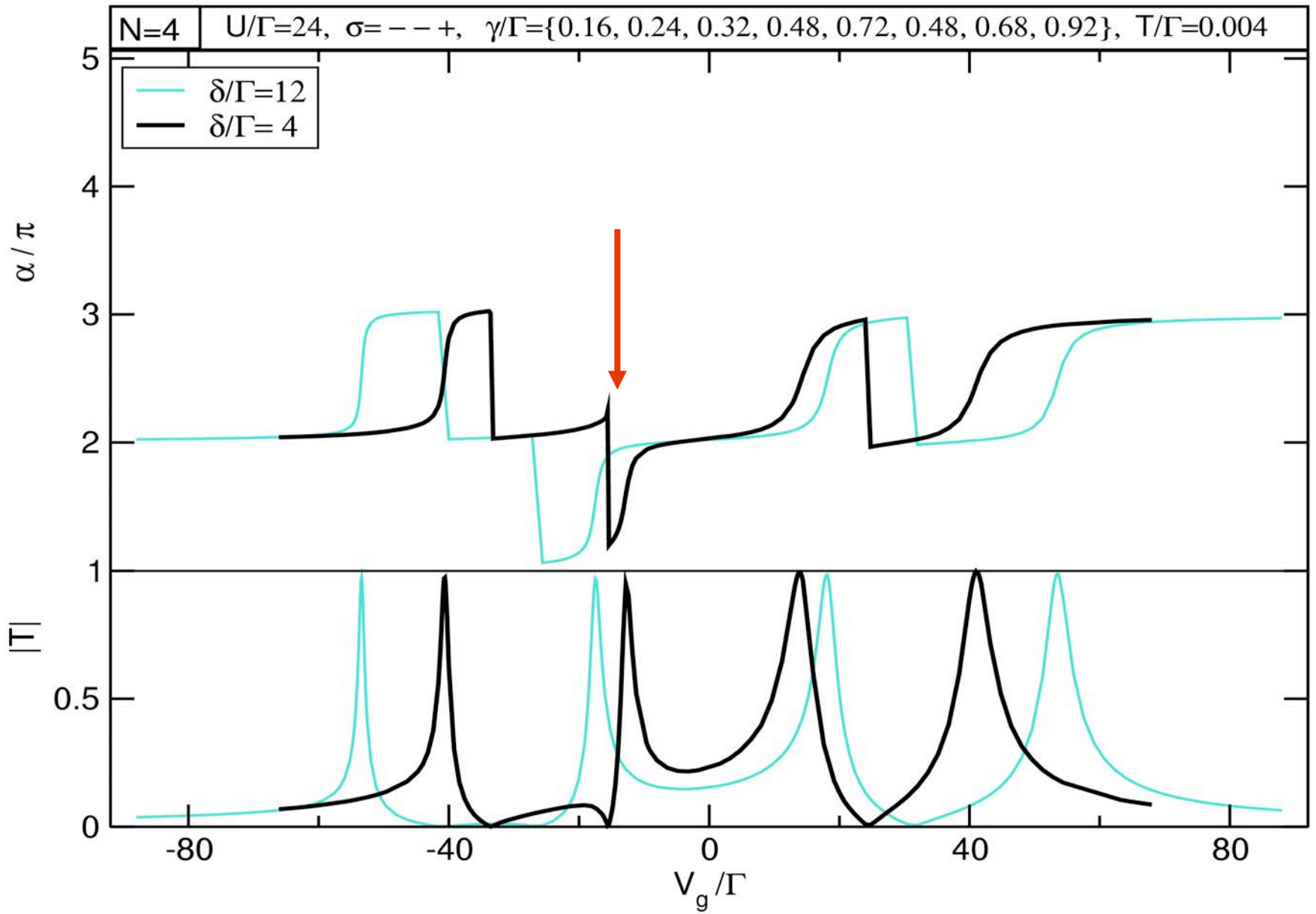
NRG: δ/Γ = 16 (crossover)



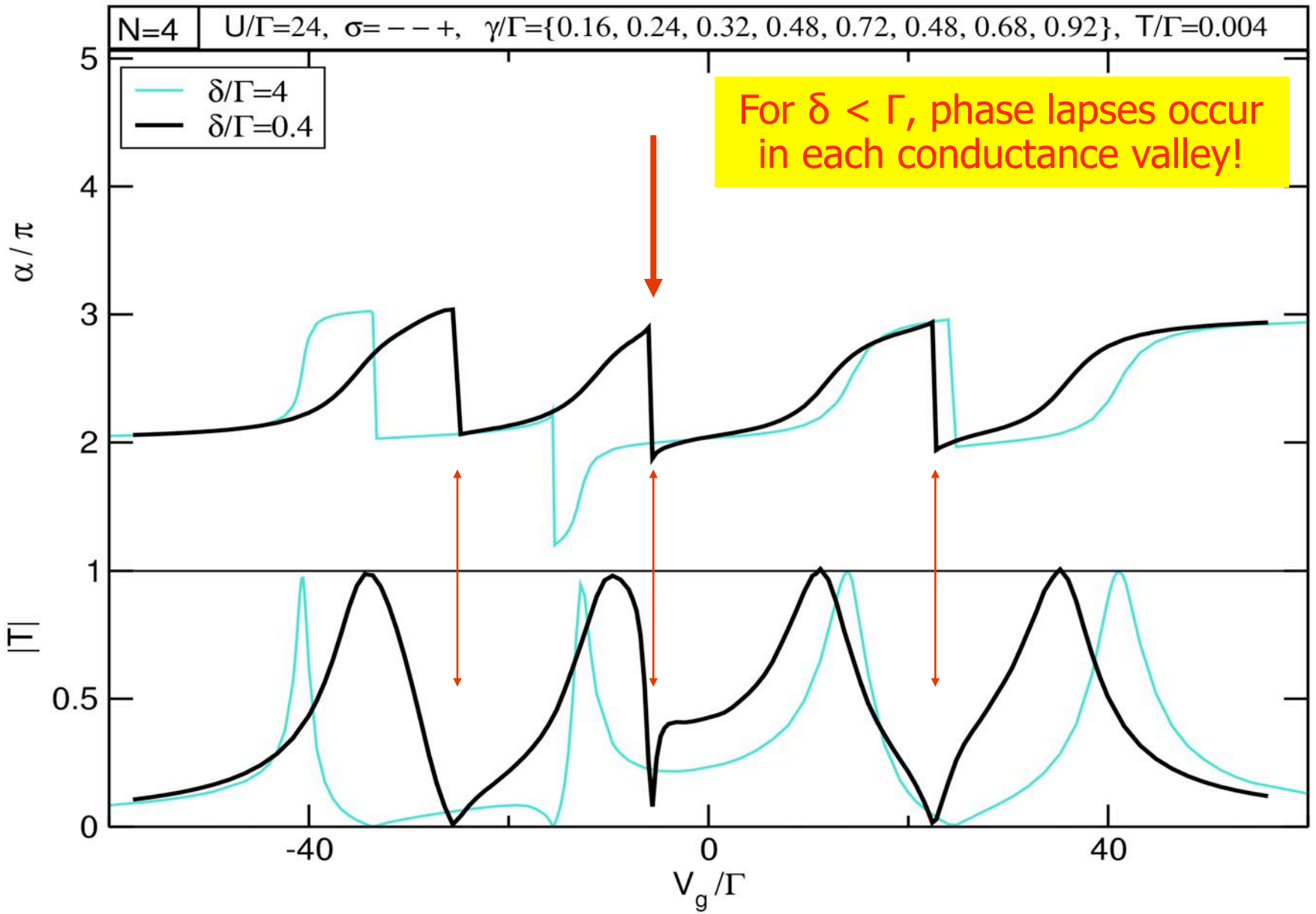
NRG: δ/Γ = 12 (crossover)



NRG: δ/Γ = 4 (crossover)

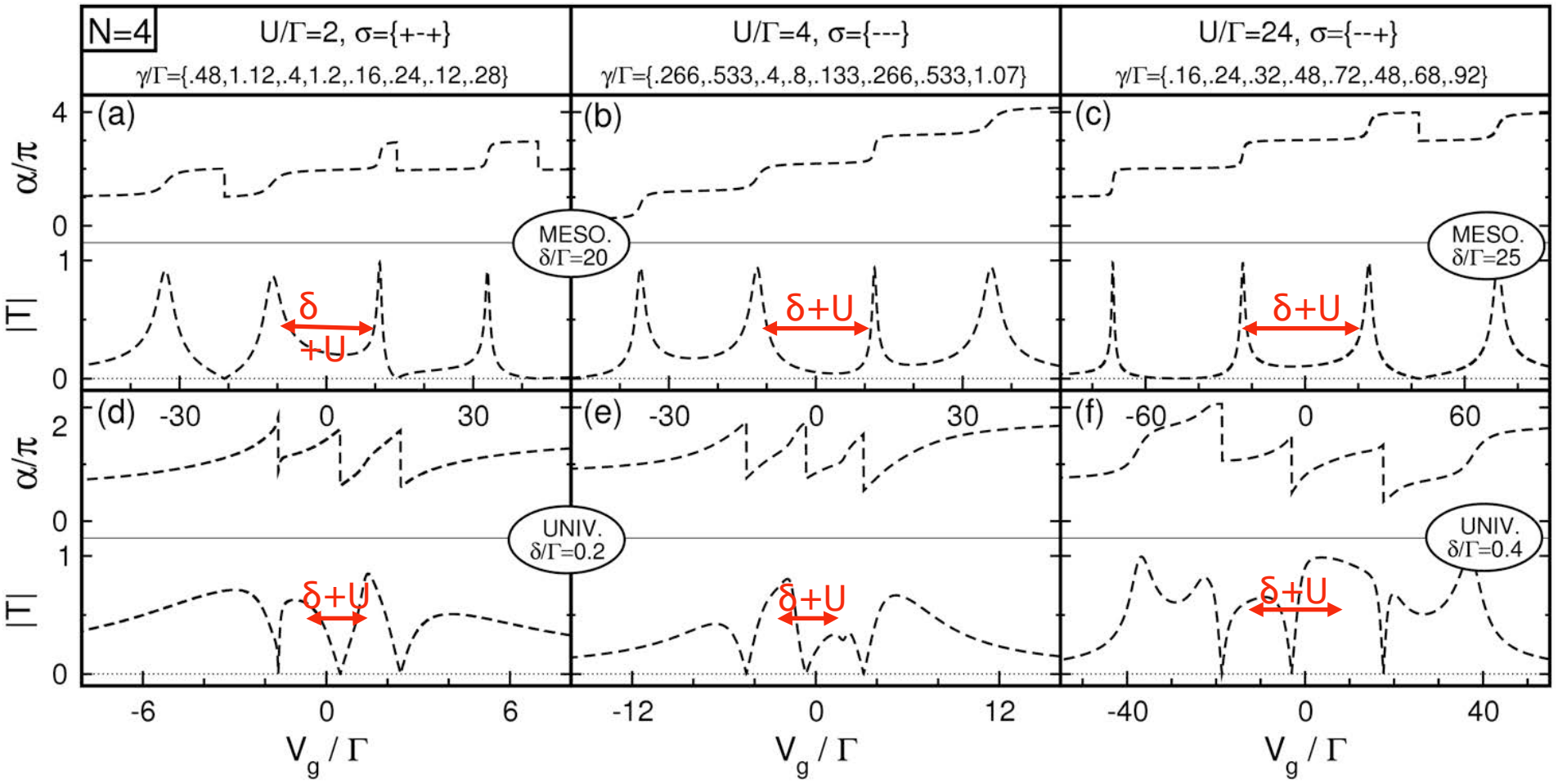


NRG: δ/Γ = 0.4 (universal behavior of phase)



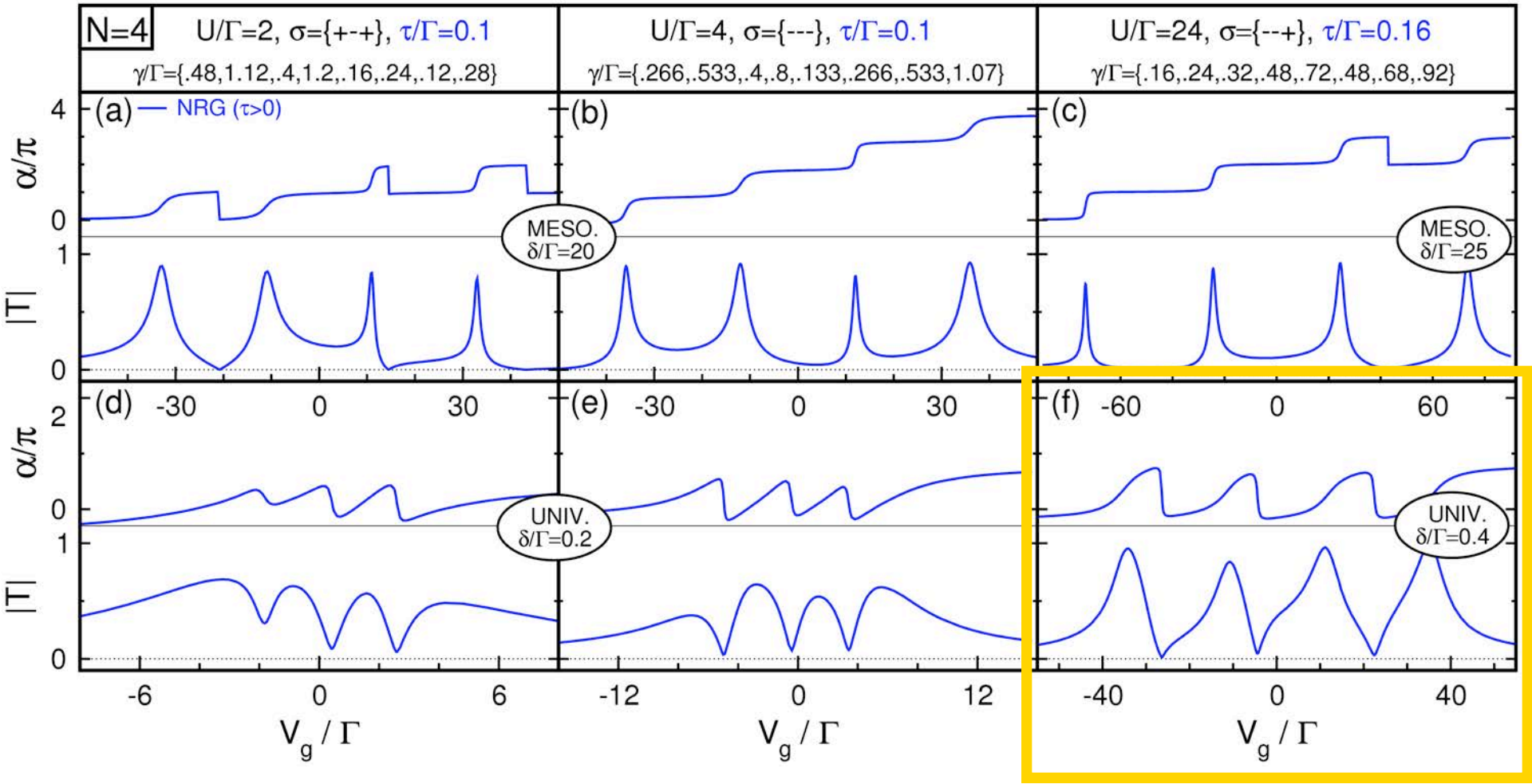
fRG at zero temperature

Using fRG, we found this to happen generically, for many different parameter sets



For δ < Γ, phase lapses occur where we want them to be!

NRG at finite temperature

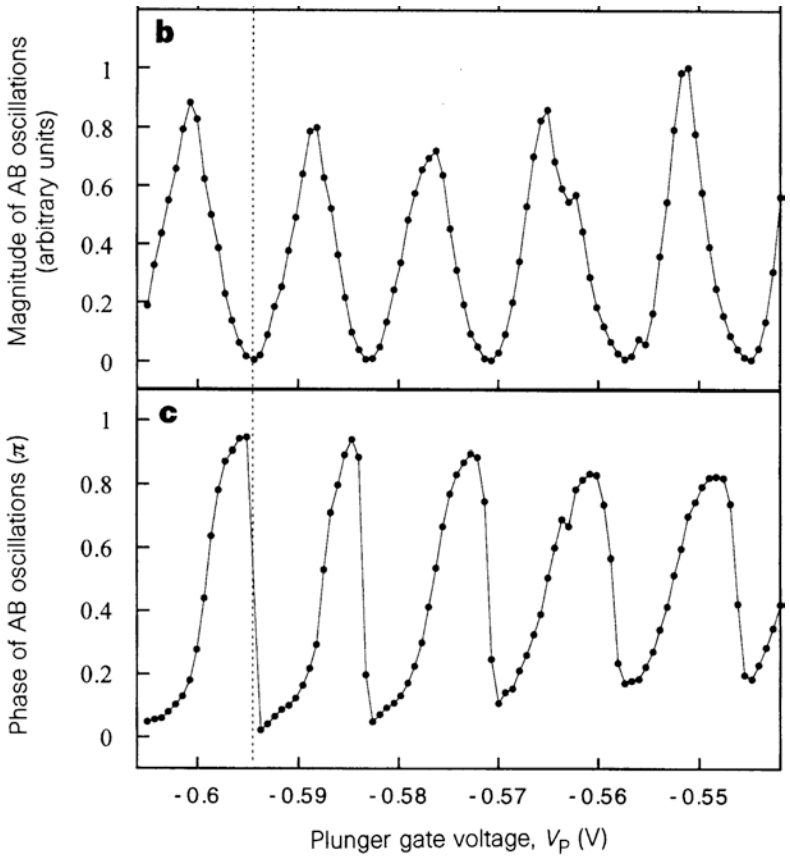


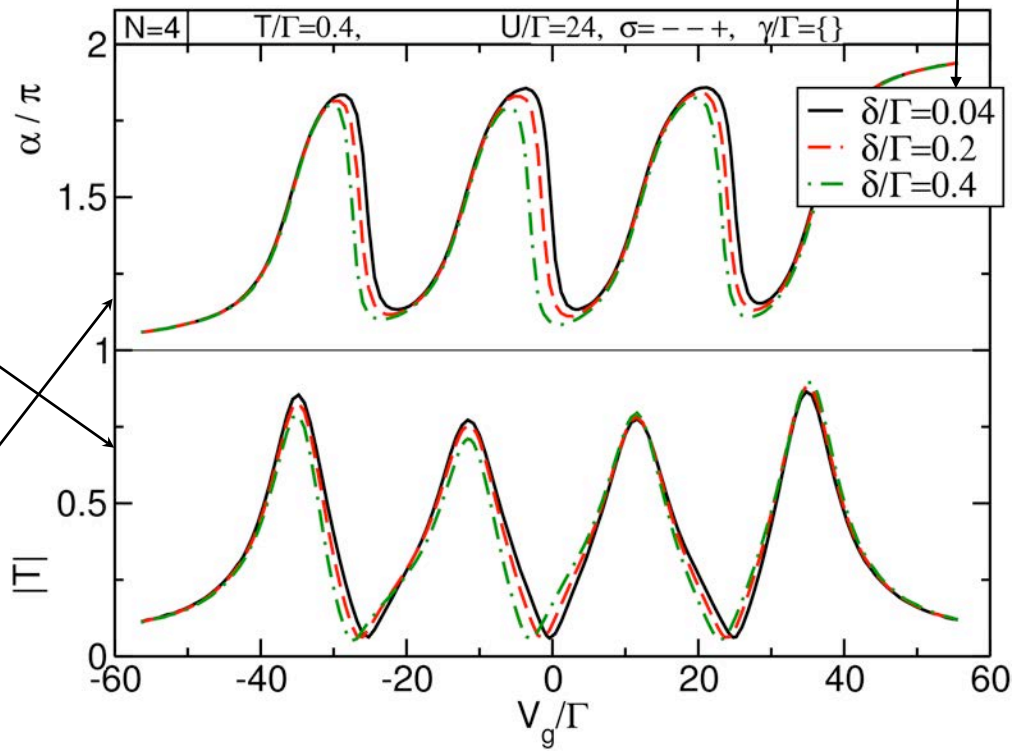
For T=0, NRG and fRG agree qualitatively very well, except for $\delta < \Gamma$, $U >> \delta$

For T≠0, sharp features features are smoothed out, results similar to exp.!!

Qualitatively, experimental trends are reproduced!

(for T>δ, the δ-dependence is weak)





But the question remains: what is the mechanism at work here?
fRG has the answers!

fRG: RG-enhanced perturbation theory in U

$U = 0$: integrate out leads:

$$\mathcal{G}^{0R}(0)^{-1}_{ij} = \underbrace{-h^0_{ij} + i\overbrace{\Delta_{ij}}^{\pi\rho\sum_\alpha t_i^\alpha t_j^\alpha}}_{\text{eigenvalues: } -\varepsilon_j + i\Gamma_j}$$

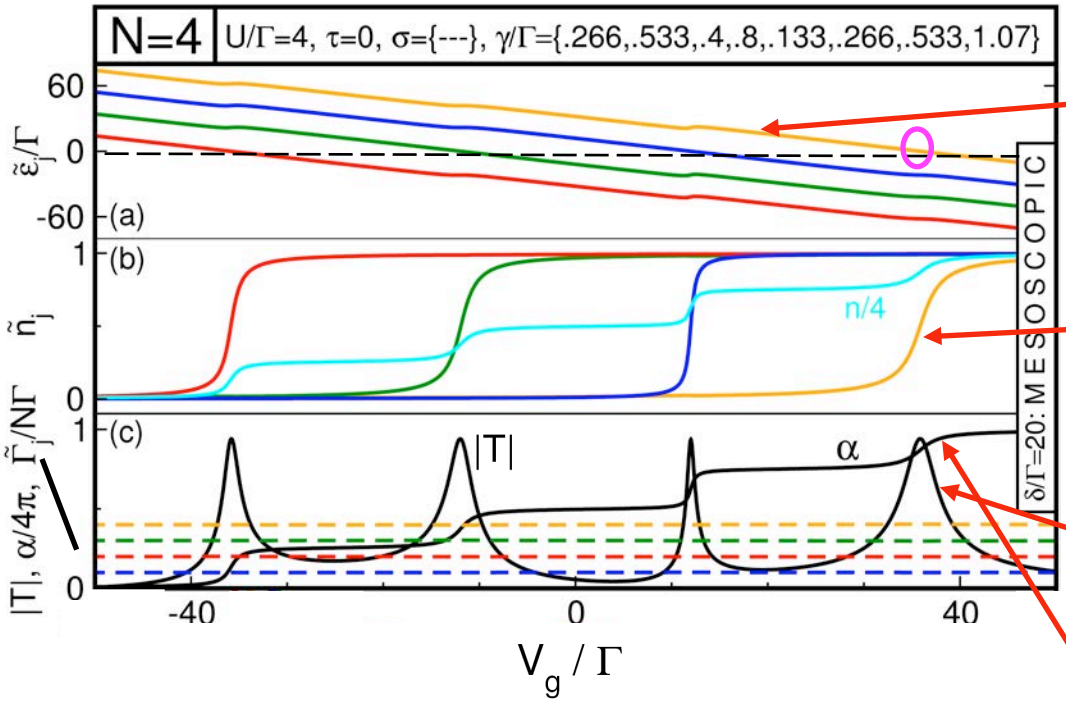
Effective level positions and widths of bare single-particle model.

$U \neq 0$: RG-improved pert. theory in U:

$$\mathcal{G}^{R}(0)^{-1}_{ij} = \underbrace{-h^0_{ij} + \Sigma_{ij} + i\Delta_{ij}}_{\text{eigenvalues: } -\tilde{\varepsilon}_j + i\tilde{\Gamma}_j}$$

Effective level positions and widths of **renormalized** single-particle model.

fRG renormalized single-particle interpretation: δ >> Γ



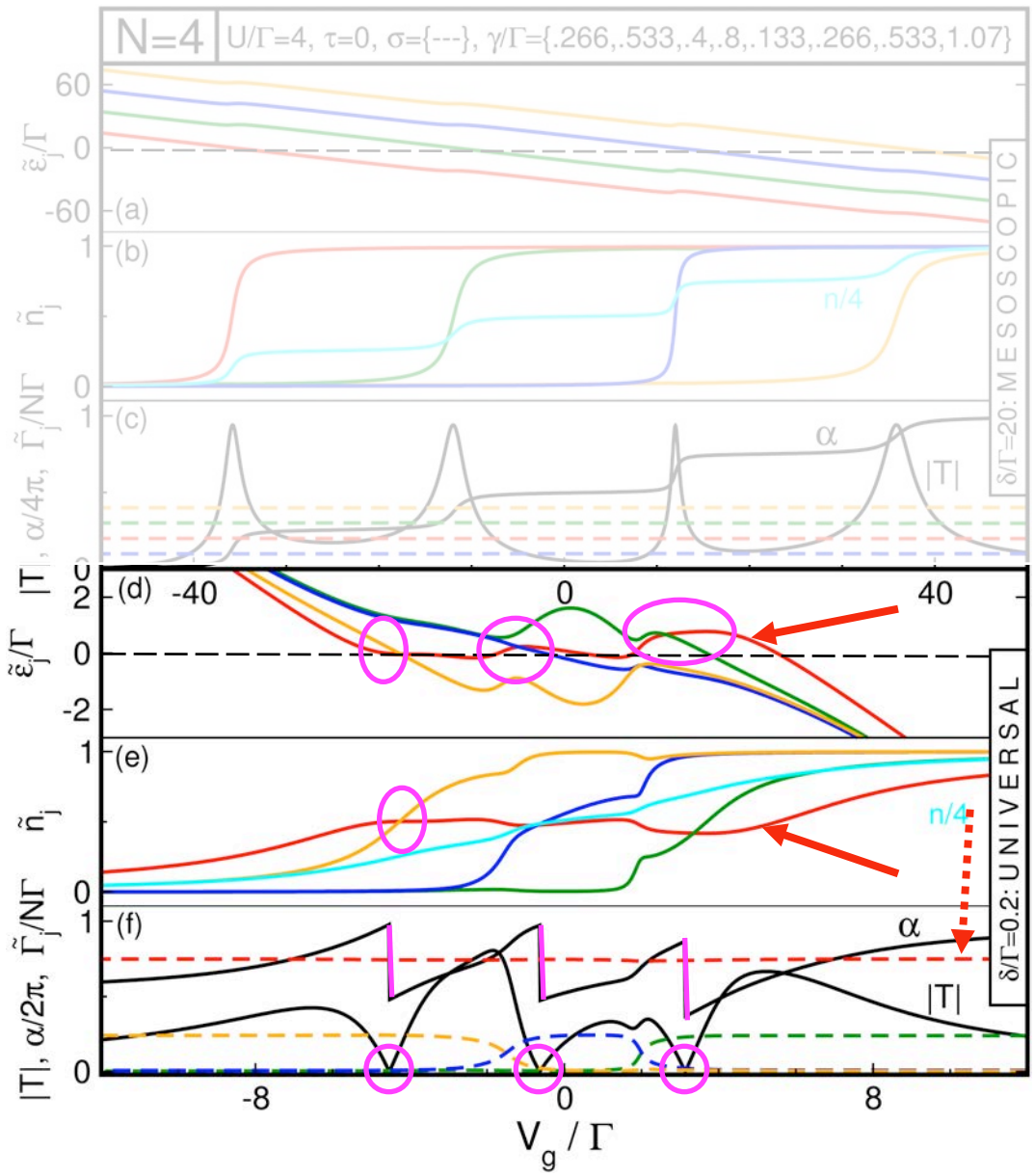
For δ >> Γ, level positions vary roughly linearly with V_g

Each time one of the narrow levels crosses the chemical potential, its occupation increases from 0 to 1.

This leads to Coulomb blockade resonances in the transmission.

The phase climbs by π across each resonance; its behavior between resonances depends on signs of matrix elements.

fRG renormalized single-particle interpretation : δ < Γ



For δ < Γ, one effective level is much broader than others.

Its energy hovers near chemical potential of leads, while its occupancy stays near ½.

Each time one of the narrow levels crosses the chemical potential of leads it also crosses the broad level.

Transmission amplitude has form

$$t_{\text{wide}} + \frac{t_{\text{narrow}}}{\varepsilon + i\tilde{\Gamma}_{\text{narrow}}}$$

leading to destructive quantum interference between broad and the narrow level: "Fano effect" !!

For time-reversal symmetry, this always causes transmission zero ("Fano antiresonance") and phase lapse.

Scenario involving broad level was studied by Silvestrov & Imry

Our scenario differs from theirs,

which assumes $U >> \Gamma_{wide}$

Origin of broad level (Dicke effect)

Berkovits, von Oppen, Kantelhardt, EPL 2004

$$\mathcal{G}^R(0)^{-1}_{ij} = \overbrace{-h^0_{ij} + \Sigma_{ij} + i\,\underbrace{\Delta_{ij}}_{\pi\rho\sum_\alpha t_i^\alpha t_j^\alpha}}^{\text{eigenvalues: } -\tilde{\varepsilon}_j + i\tilde{\Gamma}_j}$$

Rank two matrix,
has 2 nonzero eigenvalues.
(larger one dominates).

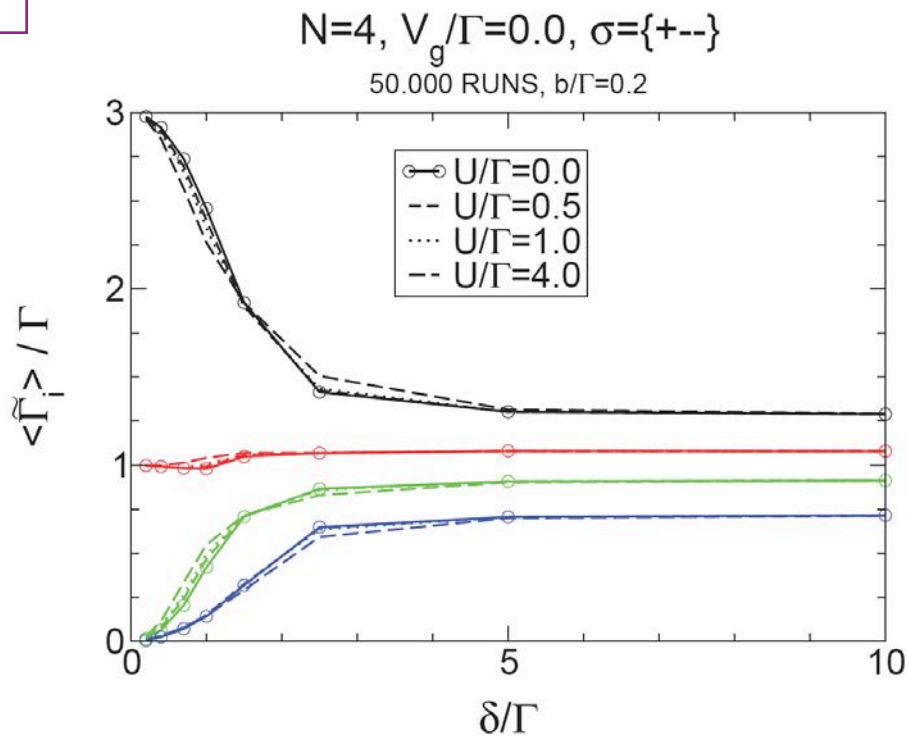
dominates if $\delta > \Gamma$

dominates if $\delta < \Gamma$

For $\delta \lesssim \Gamma$, broad level emerges automatically

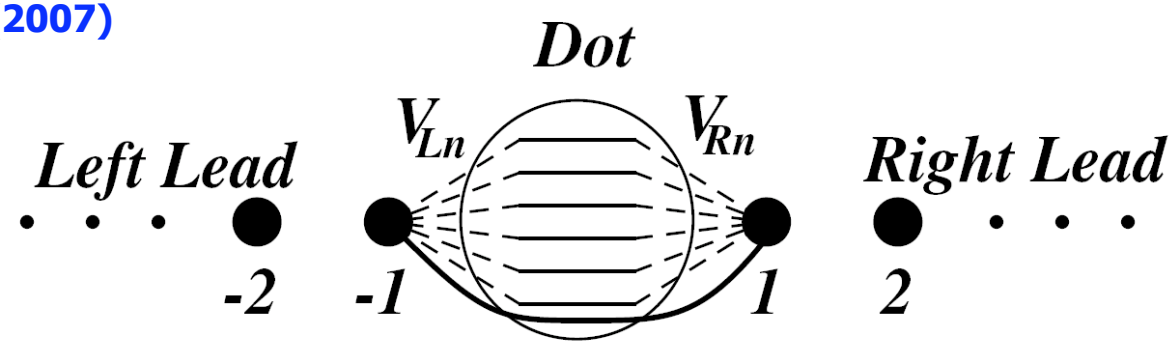
(for large N, $\tilde{\Gamma}_{max} \sim N$)

Renormalized level widths are essentially independent of U (!)



Noninteracting model with single wide, hovering level

Y. Oreg, to appear in New J. Phys. (2007)

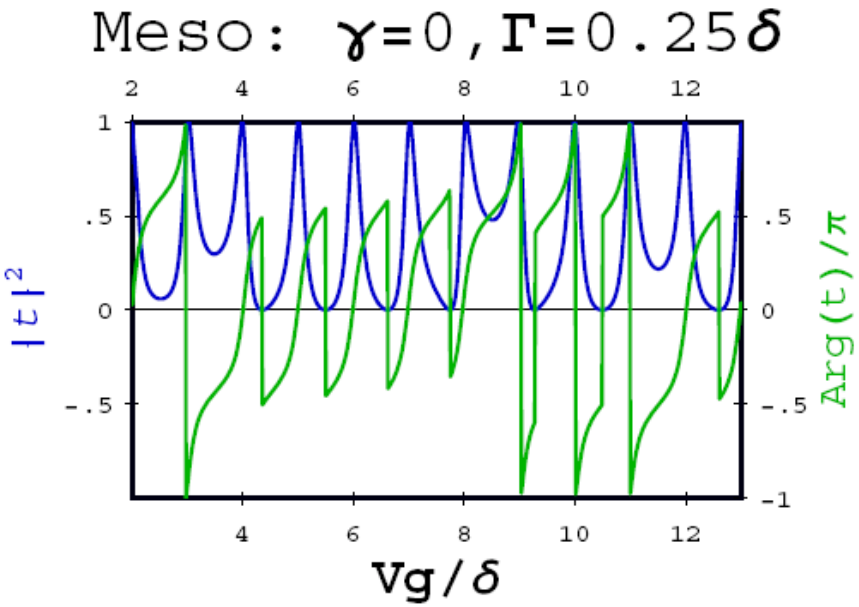


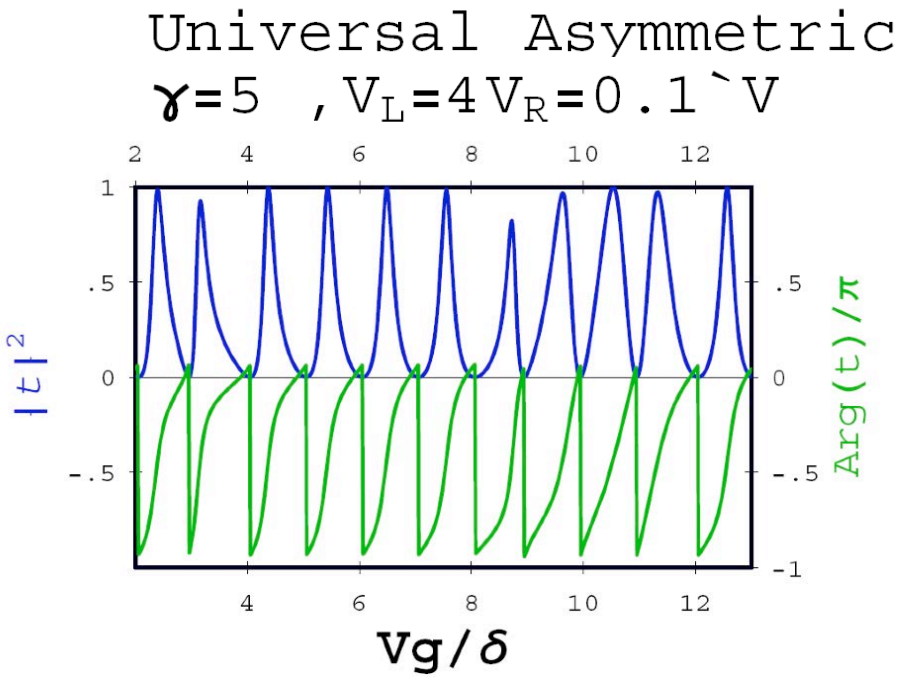
Assumptions:

No interactions: U=0

Many narrow levels, uniform spacing δ

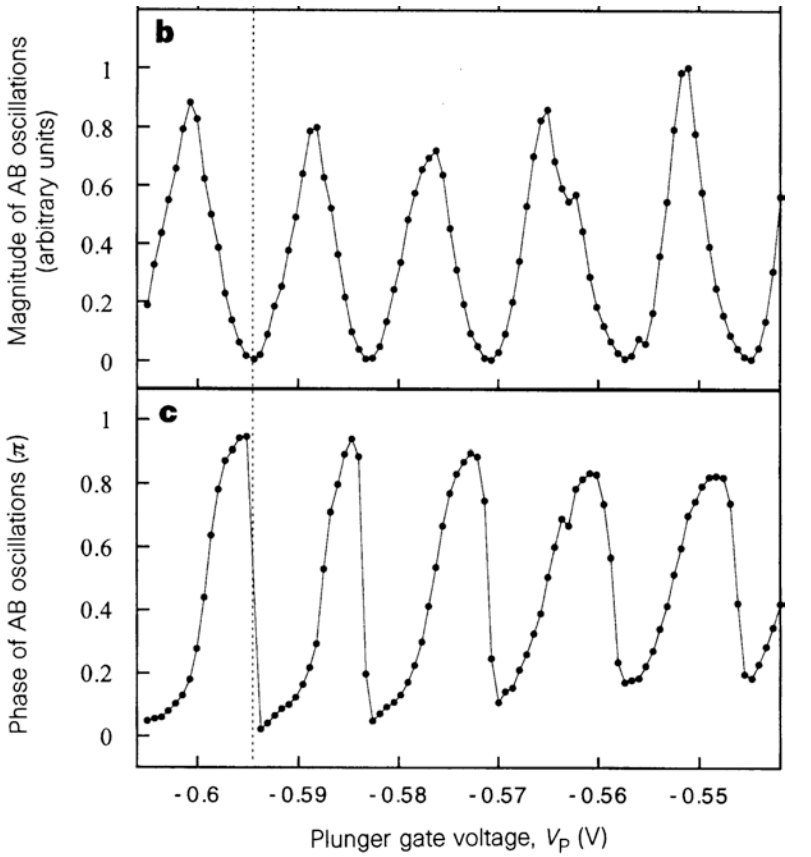
One wide, hovering level, position: $\epsilon_{wide}=0$, width: $\gamma \gg \Gamma_{narrow}$

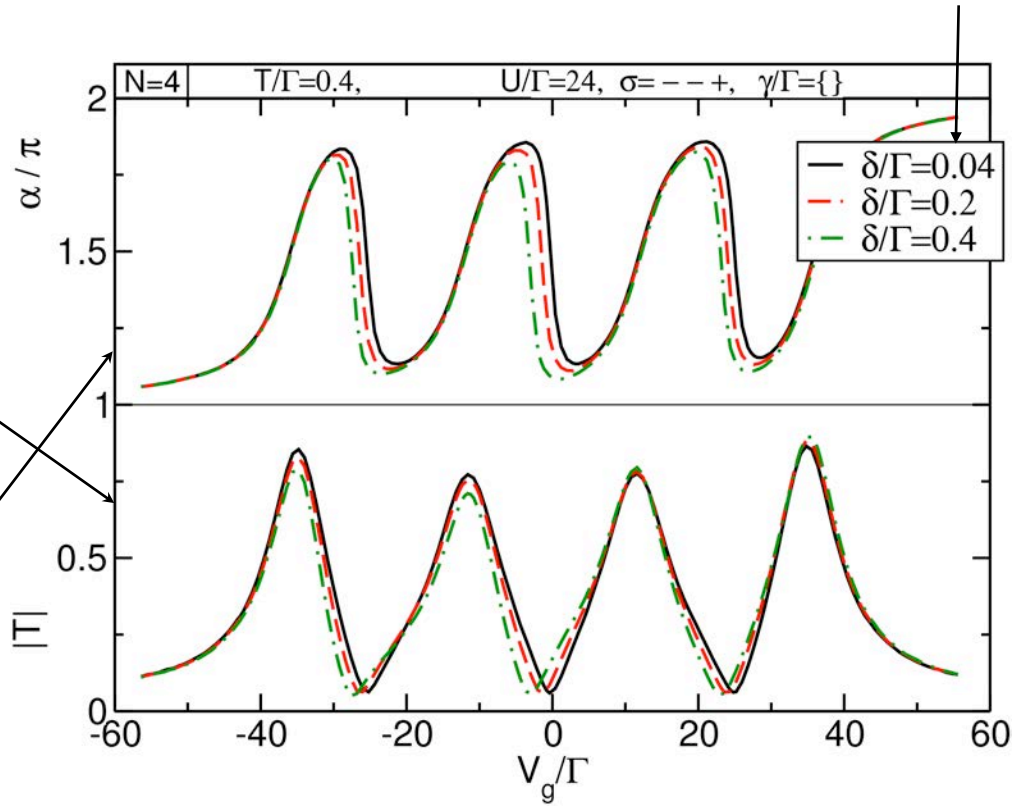




Conclusions

- Broad level emerges naturally once δ< Γ
- Broad effective level produces Fano-antiresonances with narrow levels.
- Results are in qualitative agreement with experiment.



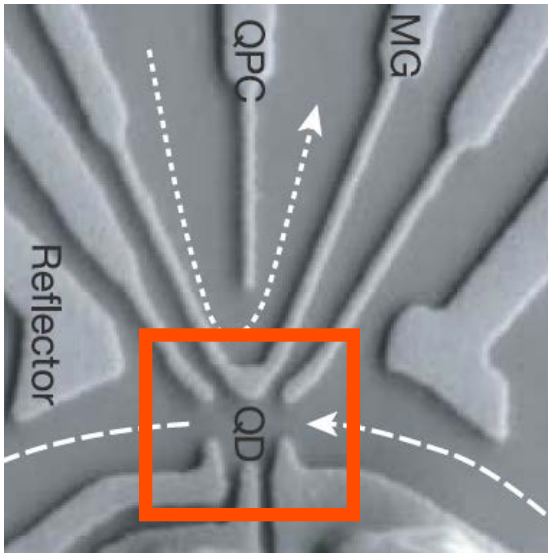


Karrasch, Hecht, Weichselbaum, von Delft, Oreg, Meden: PRL 2007, NJP 2007

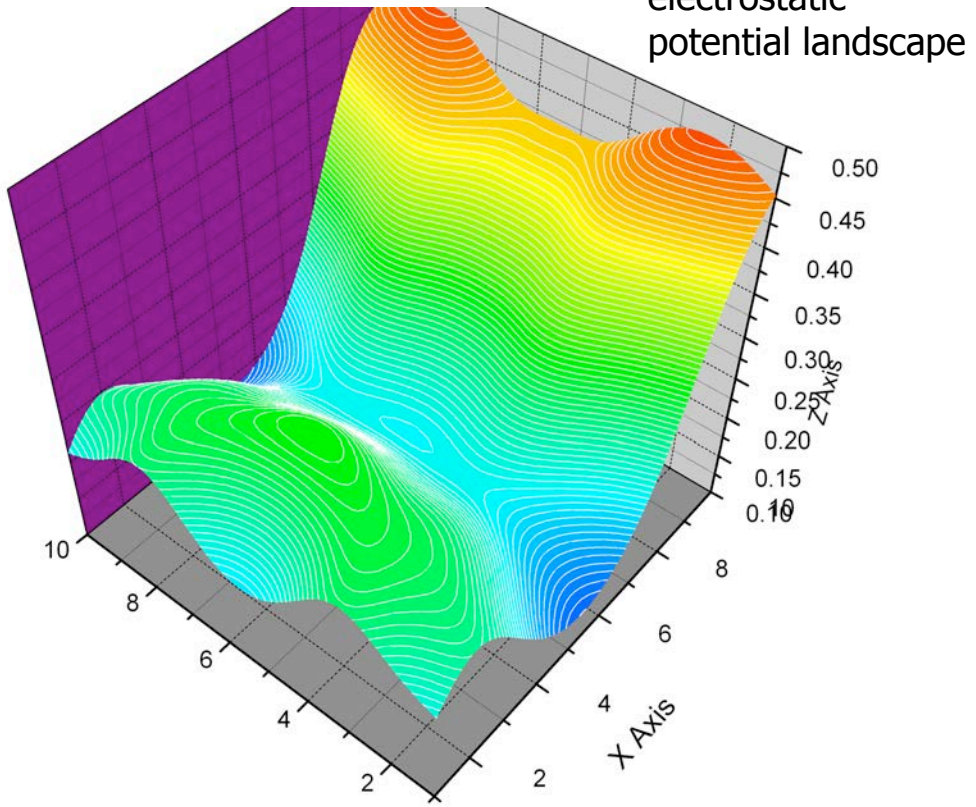
Open issues

- Study role of spin for N=3, 4 levels… (presumably not relevant in regime $\delta < \Gamma$)
- May we use bare parameters with $\delta < \Gamma$?

  Conventional wisdom says this is not generic.
- Understand better which conditions produce broad levels (geometry-dependent?)
- Find out by doing detailed modelling !



electrostatic
potential landscape



work in progress…

Thank you for your attention !

Scenario of Silvestrov & Imry

Silvestrov & Imry, PRL **85**, 2565 (2000); Phys. Rev. B **65**, 035309 (2001)

Assumed one broad level (backed up by model calculations for chaotic dots)

Assumed $U >> \Gamma_{wide}$

Showed that this leads to succession of (strong) occupation inversions:

wide level fills up, gets emptied into narrow level, fills up again, etc.

Pointed out that a phase lapse occurs at each occupation inversion,

implying universal succession of



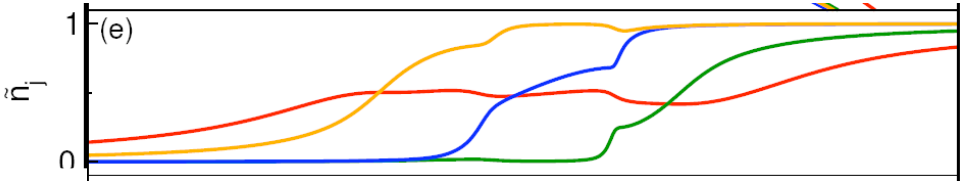
How does our work differ from theirs?

we do not assume a wide level: it appears generically for $\delta < \Gamma$ (Dicke effect)

condition $U >> \Gamma_{wide}$ is not needed to find universal phase lapses

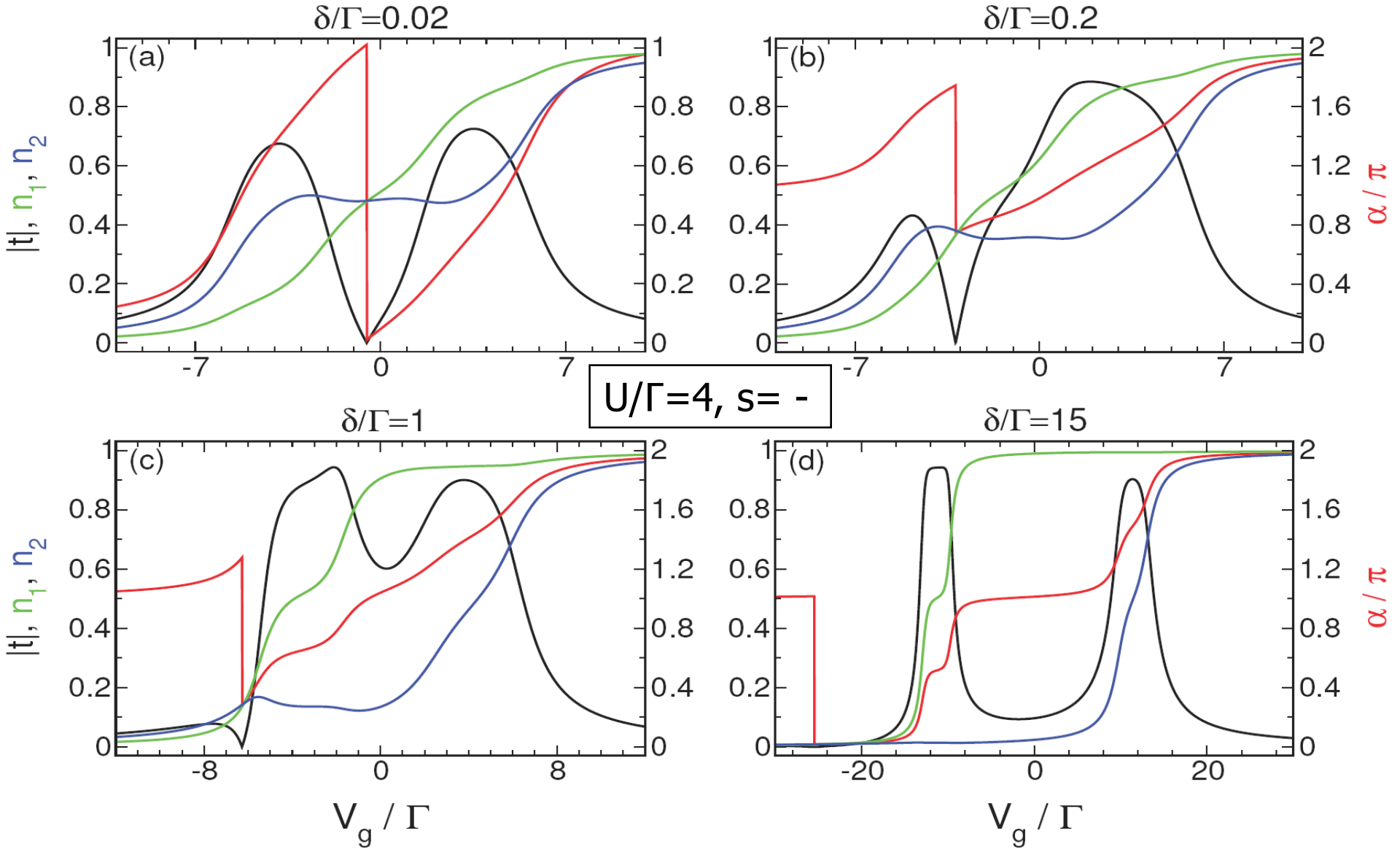
whether (strong) occupation inversion occurs or not is irrelevant (it is a side effect).

essential physics is due to Fano interference of broad with narrow level



Role of spin?

Presumably not relevant in regime $\delta < \Gamma$, where no Kondo physics occurs



fRG: RG-enhanced perturbation theory in U

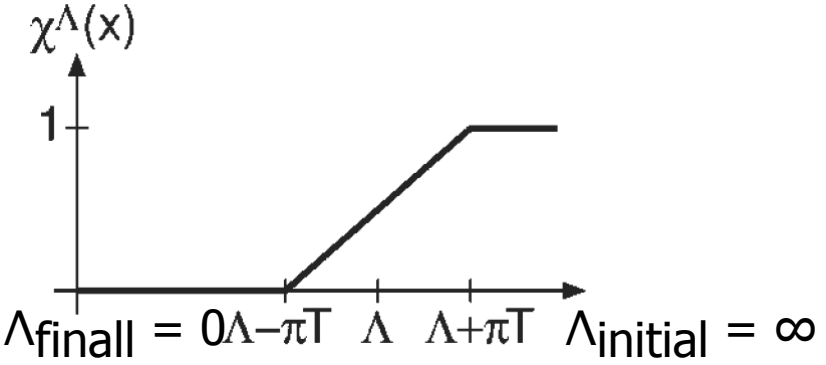
$U = 0$: integrate out leads $\qquad$ $U \neq 0$: RG-improved pert. theory

$$\mathcal{G}^{0R}(0)_{ij}^{-1} = -h_{ij}^{0} + i\,\underbrace{\Delta_{ij}}_{\pi\rho\sum_\alpha t_i^\alpha t_j^\alpha} \quad \xrightarrow{\text{RG}} \quad \mathcal{G}^{R}(0)_{ij}^{-1} = \underbrace{-h_{ij}^{0} + \Sigma_{ij} + i\Delta_{ij}}_{\text{eigenvalues: } -\tilde{\varepsilon}_j + i\tilde{\Gamma}_j}$$

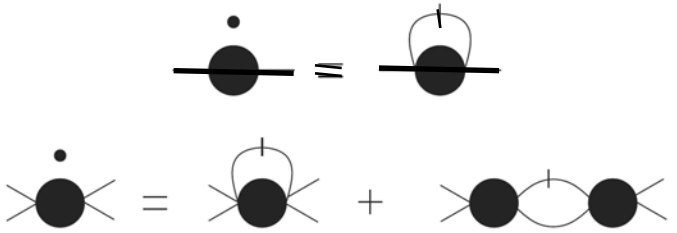
Effective level positions and widths of renormalized single-particle model.

RG introduces infrared cutoff Λ, and reduces it to zero in infinitesimal steps

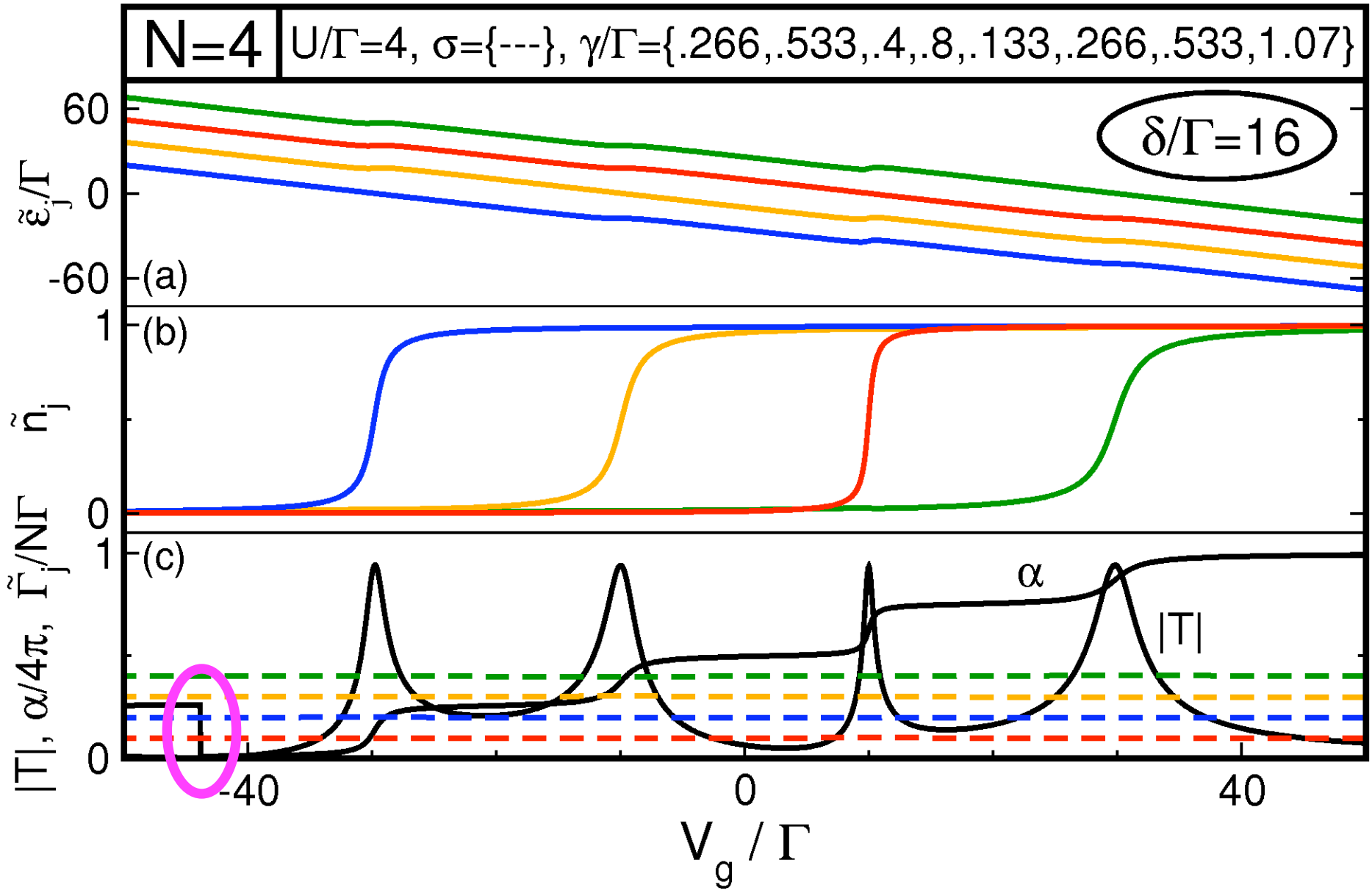
$$\mathcal{G}_0^{\Lambda}(i\omega_n) \;=\; \chi^{\Lambda}(\omega_n)\mathcal{G}_0(i\omega_n)$$



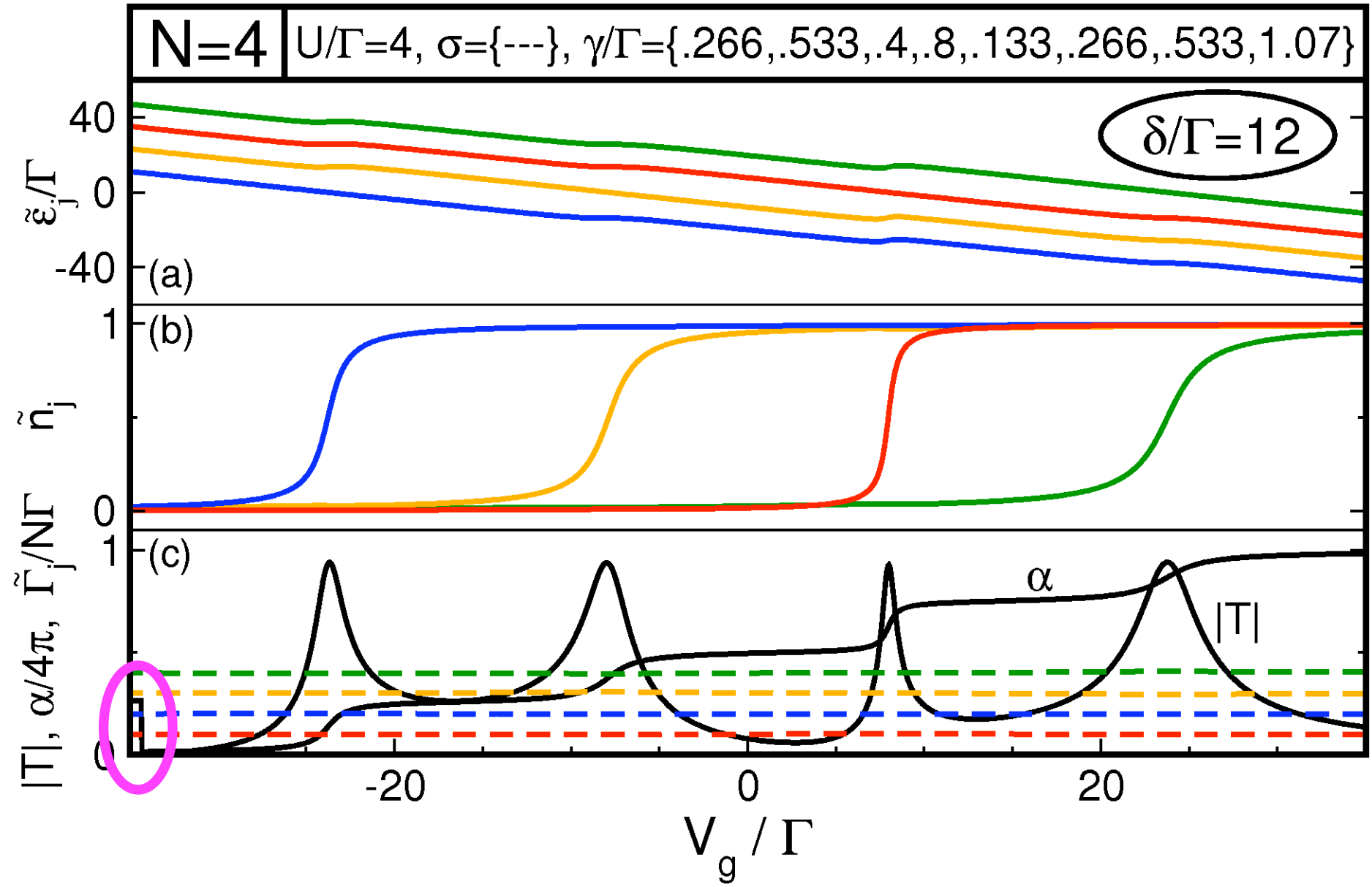
$$\mathcal{G}^{\Lambda} \;=\; \Big[\big[\mathcal{G}^{0,\Lambda}\big]^{-1} - \Sigma^{\Lambda}\Big]^{-1} \;-\; \mathcal{G}^{\Lambda}\,\partial_{\Lambda}\big[\mathcal{G}^{0,\Lambda}\big]^{-1}\mathcal{G}^{\Lambda}$$



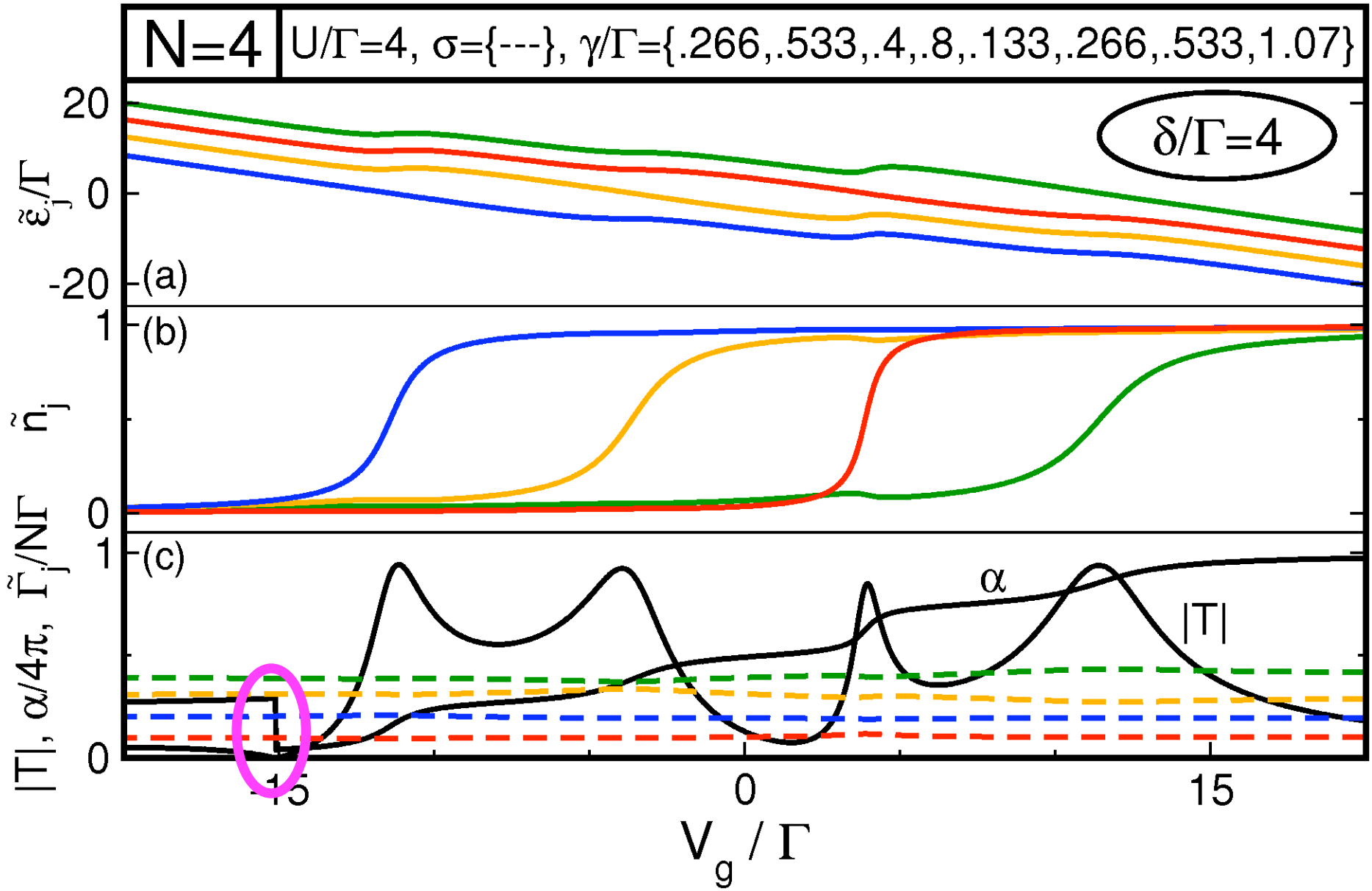
fRG: emergence of broad level



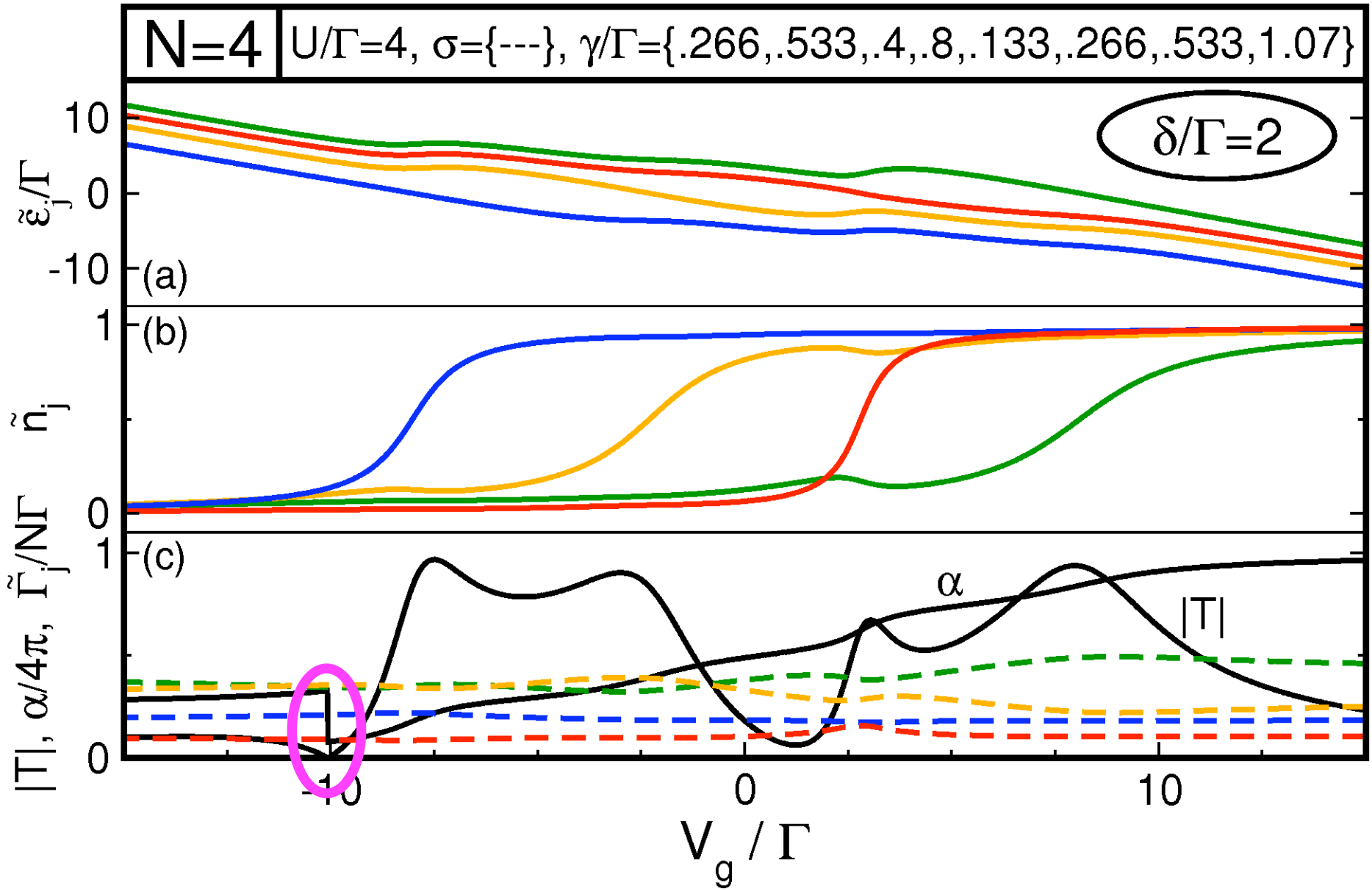
fRG: emergence of broad level



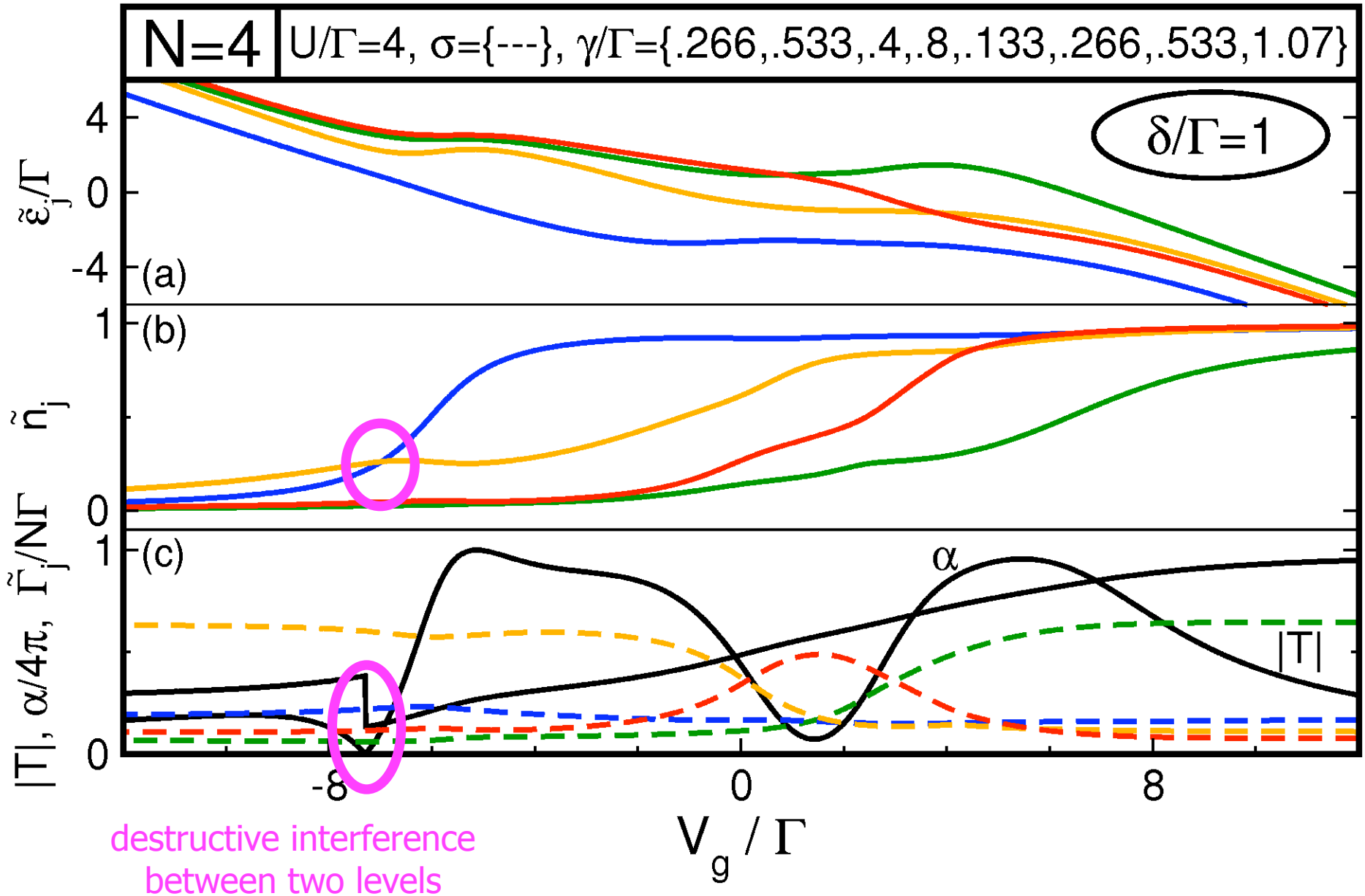
fRG: emergence of broad level



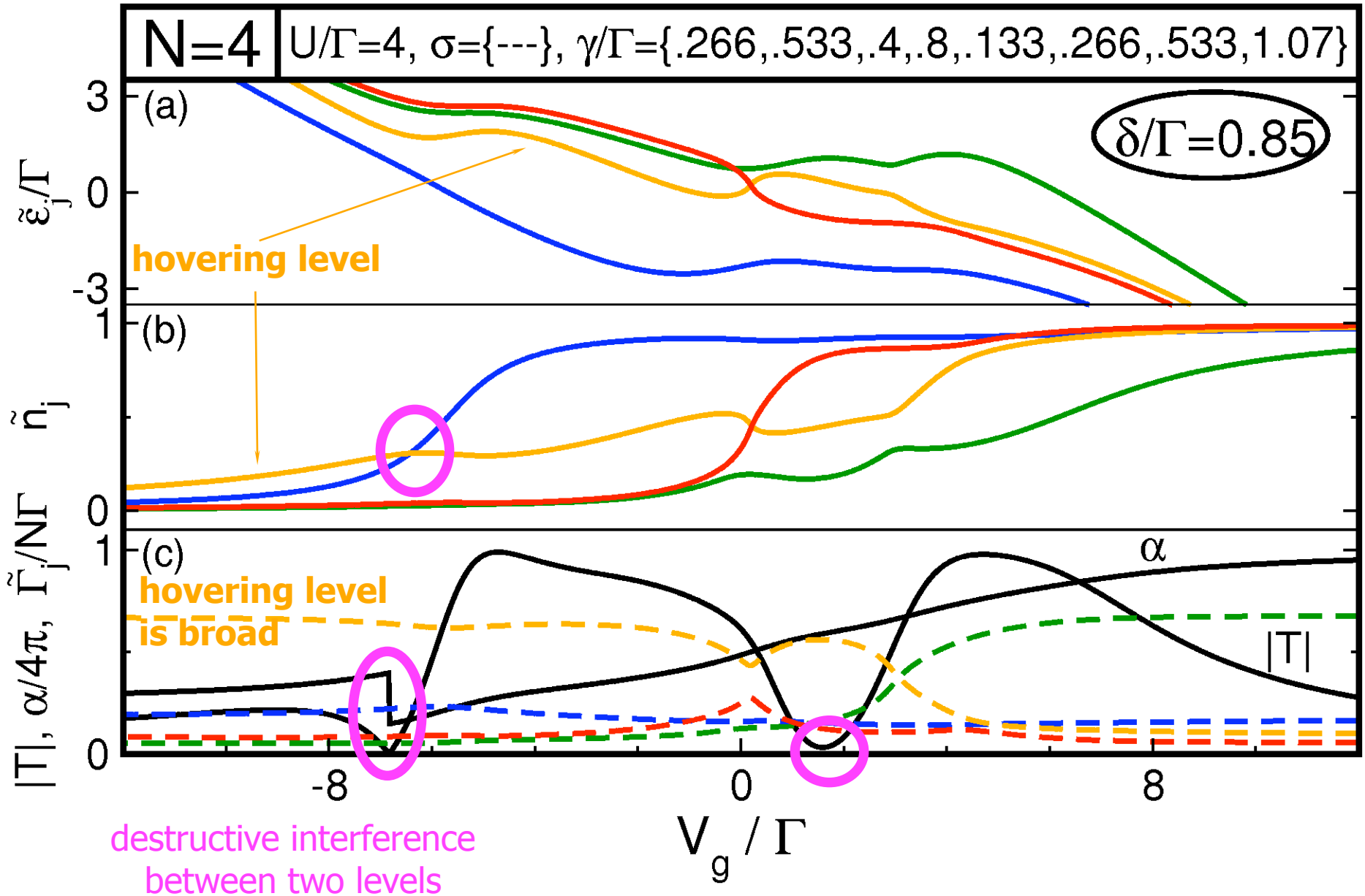
fRG: emergence of broad level



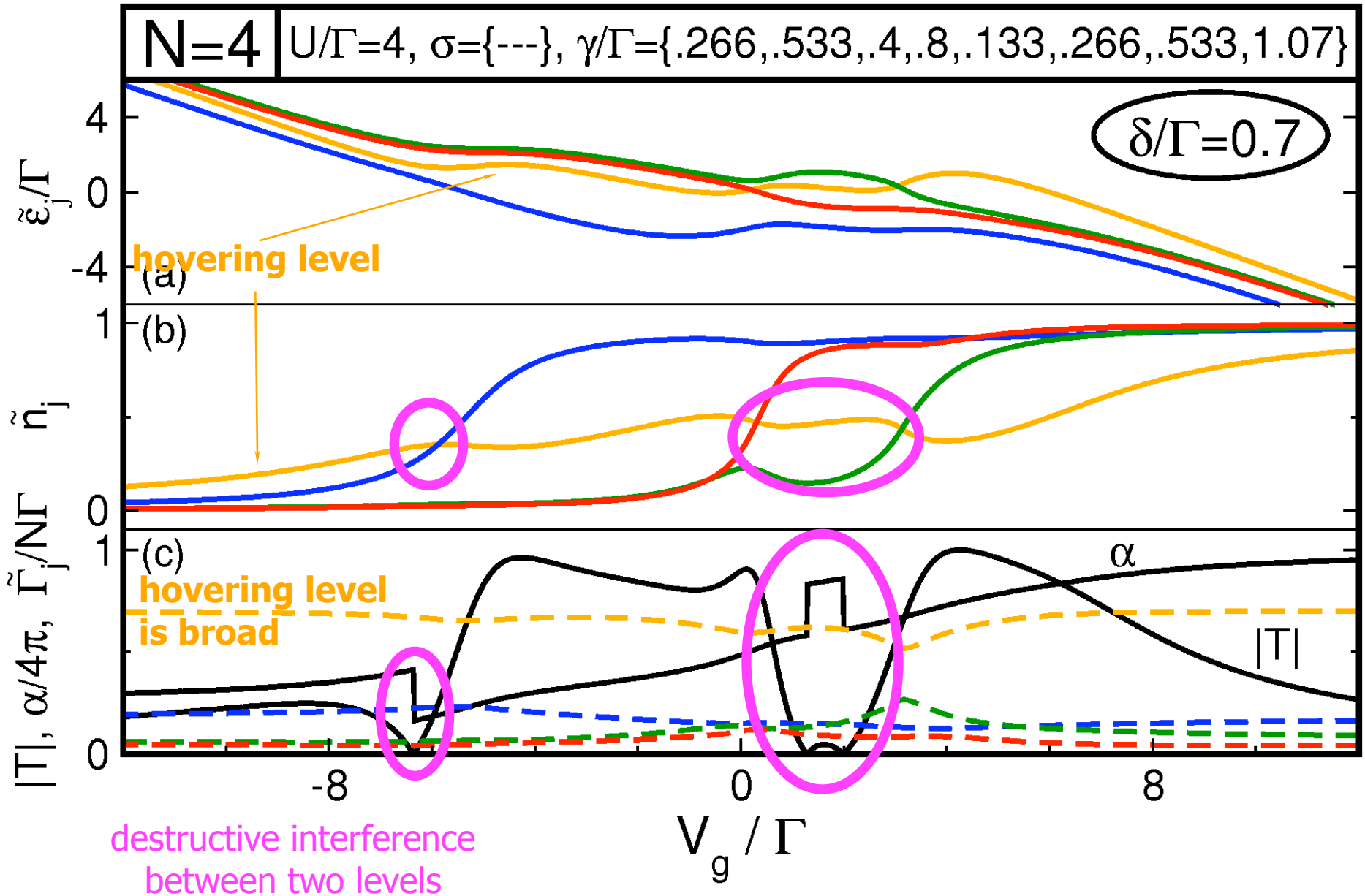
fRG: emergence of broad level



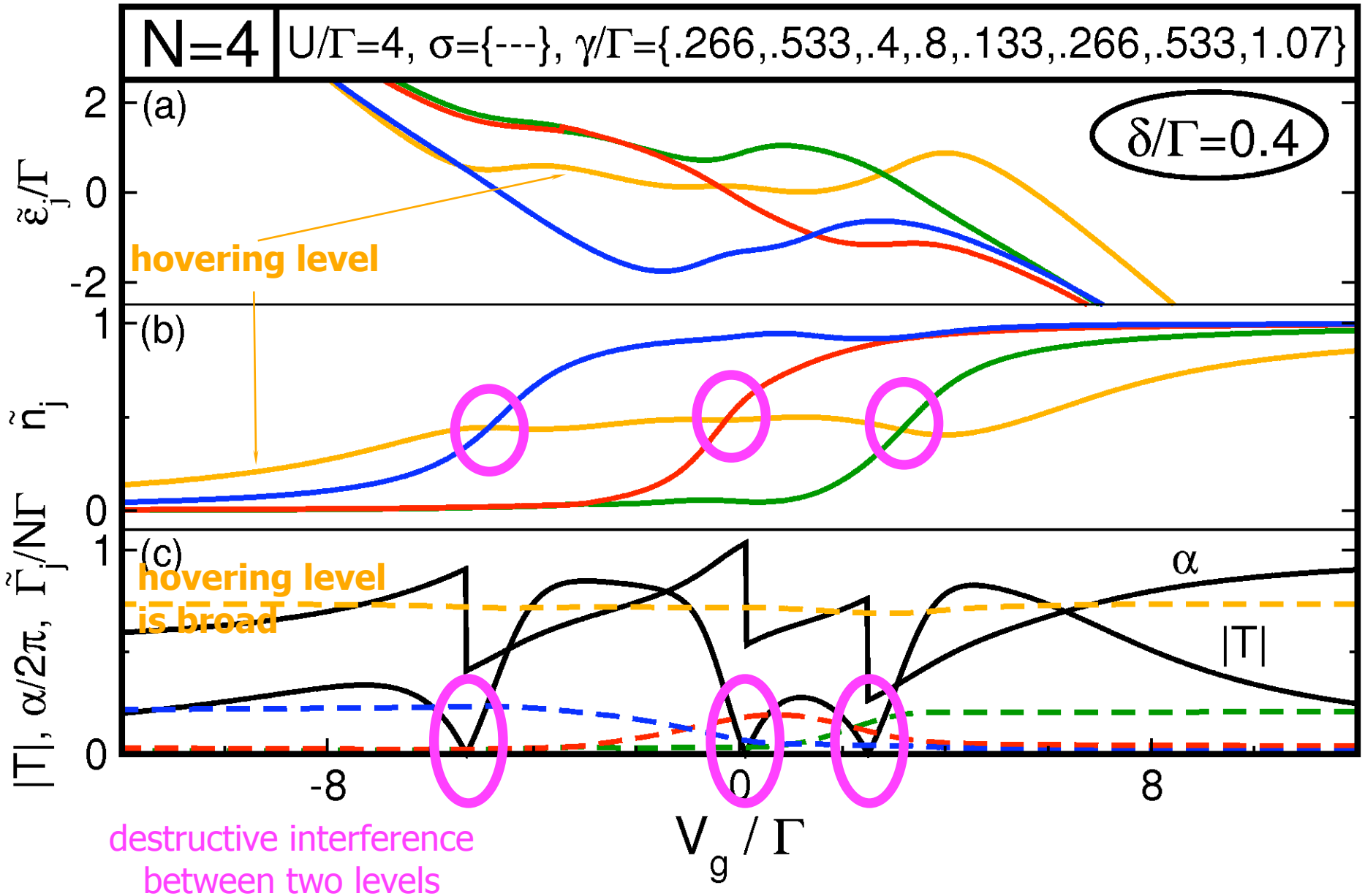
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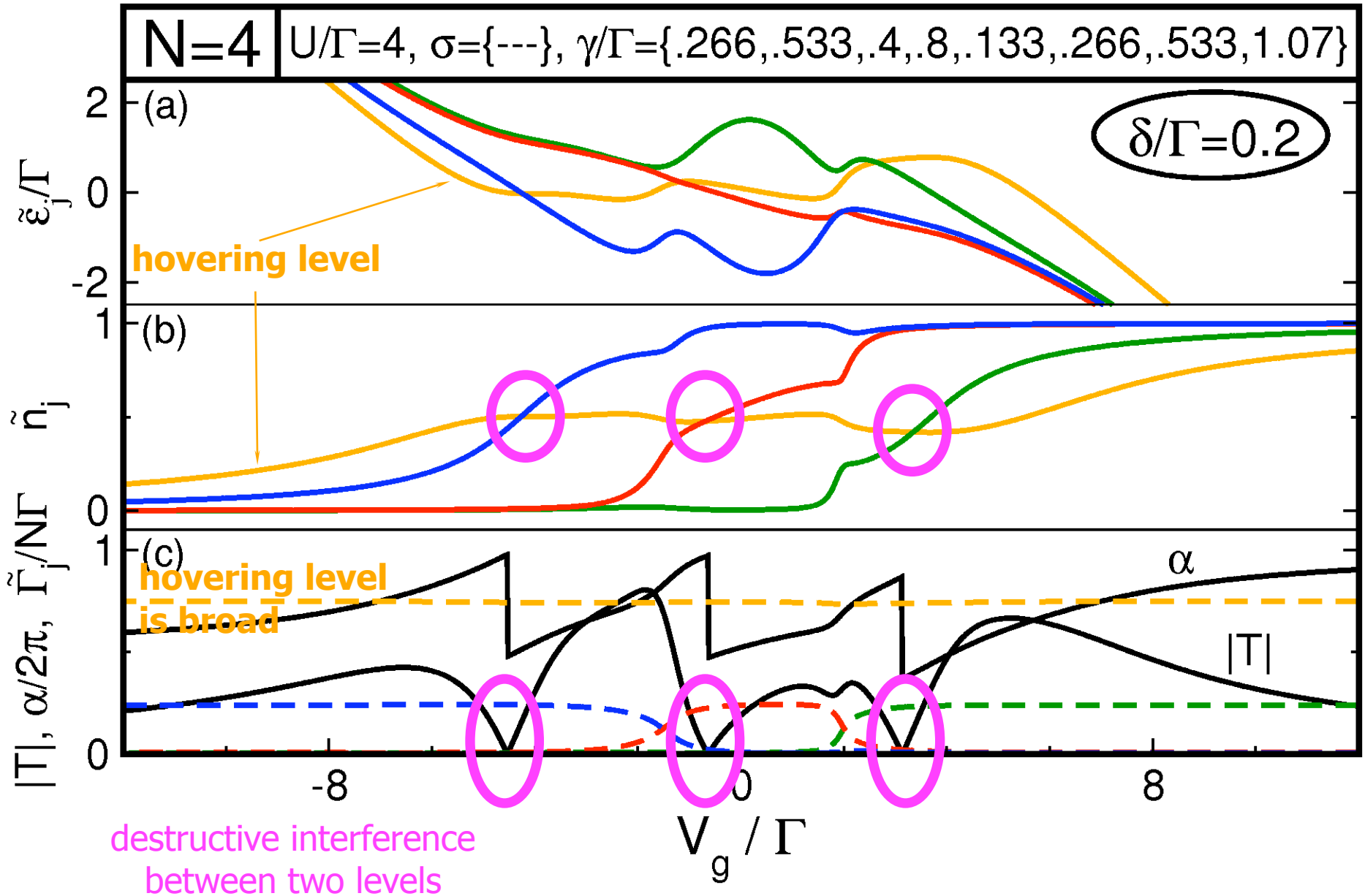
fRG: emergence of broad level



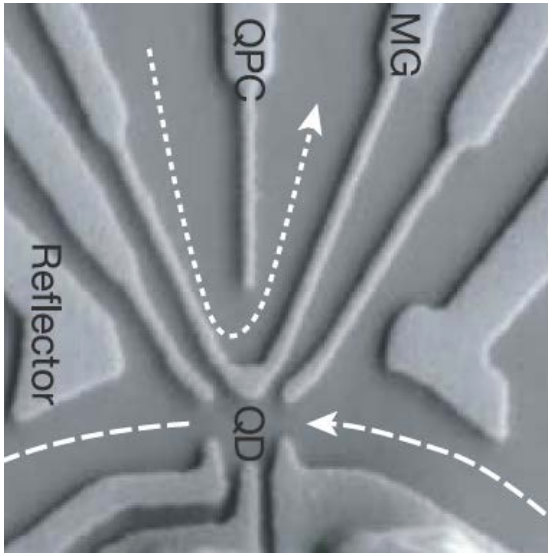
fRG: emergence of broad level

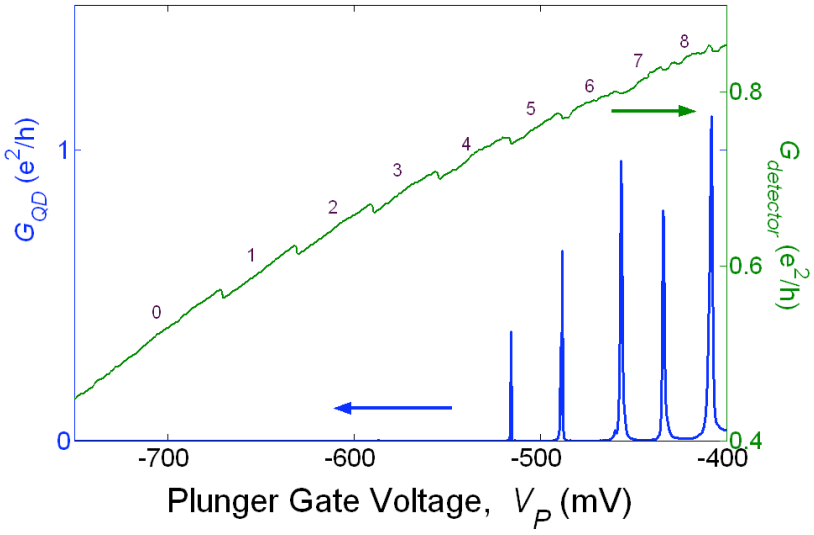


fRG: emergence of broad level



Charge detection by point contact

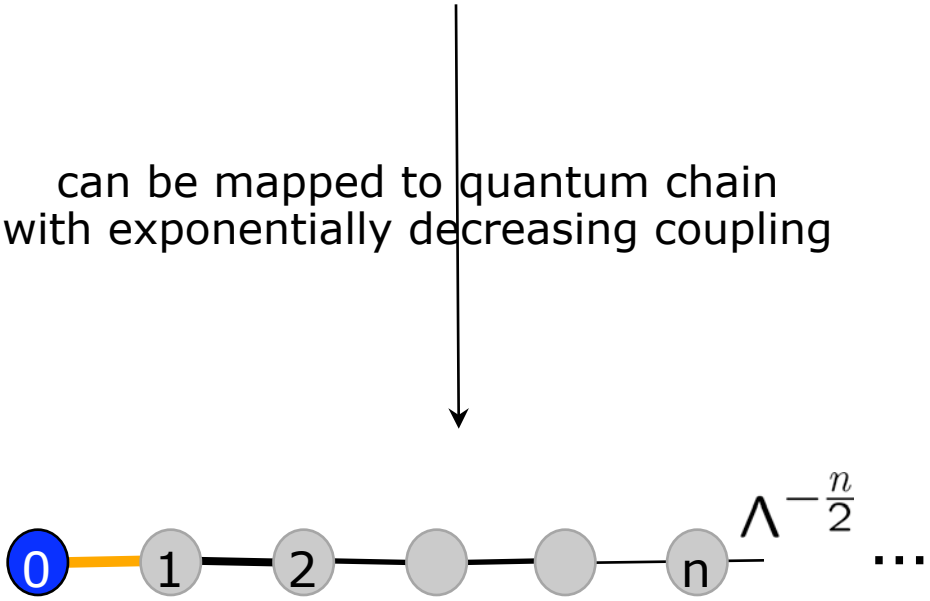


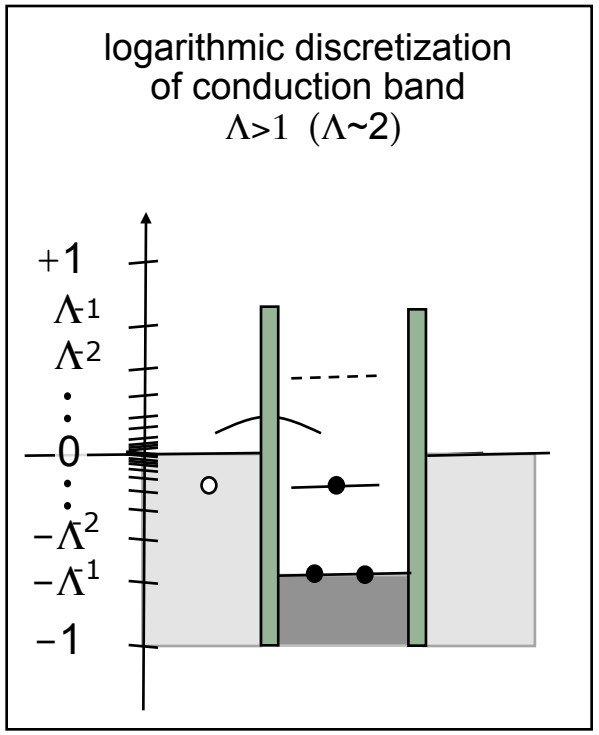


NRG: numerical diagonalization method

$$H = H_{\text{dot}} + \sum_{\mu j} \int_{-D}^{D} d\epsilon\, t_j^{\mu} \left(a_{\epsilon\mu}^{\dagger} d_j + d_j^{\dagger} a_{\epsilon\mu} \right) + \sum_{\mu} \int_{-D}^{D} d\epsilon\, \epsilon\, a_{\epsilon\mu}^{\dagger} a_{\epsilon\mu}$$

can be mapped to quantum chain
with exponentially decreasing coupling





$$H = H_{\text{dot}} + \sum_{\mu j} t_j^{\mu} \left(f_{0\mu}^{\dagger} d_j + d_j^{\dagger} f_{0\mu} \right) + \sum_{\mu} \sum_{n=0}^{\infty} \Lambda^{-\frac{n}{2}} \left(f_{n\mu}^{\dagger} f_{n+1,\mu} + f_{n+1,\mu}^{\dagger} f_{n\mu} \right)$$

Number parity in superconductors and the ν=5/2 fractional quantized Hall state

by Bertrand I. Halperin
Harvard University

Talk in honor of Vinay Ambegoakar on the occasion of his retirement. Ithaca, June 16, 2007

Number Parity in Superconductors

Dependence of free energy, other quantities, on whether the number of electrons in a small sample is even or odd

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PHYSICAL REVIEW B VOLUME 50, NUMBER 2 1 JULY 1994-II

BCS superconductivity with fixed number parity

Boldizsár Jankó, Anders Smith,* and Vinay Ambegaokar
Laboratory of Atomic and Solid State Physics, Cornell University, Ithaca, New York 14850-2501
(Received 28 February 1994)

VOLUME 75, NUMBER 6 PHYSICAL REVIEW LETTERS 7 AUGUST 1995

Parity Fluctuations between Coulomb Blockaded Superconducting Islands

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(Received 14 October 1994)

Moore-Read "Pfaffian" State

Moore and Read (1991) proposed a novel trial wave function as a model to explain the observed even-denominator quantized Hall state at filling fraction 5/2.

Elementary charged excitations have charge ± e/4 and obey "non-abelian statistics".

State is related mathematically to a superconductor with p_x+ip_y pairing.

The ground state energies of these states have, in common a peculiar dependence on number parity. This is important for a proposed experiment to verify the Moore-Read conjecture for the $\nu = 5/2$ state.

Outline

What do we mean by Non-Abelian statistics? How do they work in the Moore-Read "Pfaffian" state?

Description of the Pfaffian state as a p-wave BCS superconductor of "composite fermions". $\Delta \sim (p_x + ip_y)$.

Properties of a $p_x + ip_y$ superconductor.

Our proposed experiments and their predicted peculiar results for the Moore-Read state. Relation to number parity.

Non-abelian statistics for Moore-Read 5/2 state

Consider a system containing 2N localized quasiparticles, far from each other and far from boundaries. Then there exist $M=2^{N-1}$ orthogonal degenerate ground states, which cannot be distinguished from each other by any local measurement.

Moving various quasiparticles around each other and returning them to their original positions, or interchanging quasiparticles, can lead to a nontrivial unitary transformation of the ground states, which depends on the order in which the winding is performed. (Unitary matrix depends on the topology of the braiding of the world lines of the quasiparticles. Matrices form a representation of the braid group).

If two quasiparticles come close together, degeneracy is broken; but energy splittings fall off exponentially with separation.

Topological quantum computation

Non-abelian quasiparticles may be useful for "topological quantum computation".

[Kitaev, quant-ph/9707021; Freedman, Larson, Wang, Commun. Math Phys (2002); Bonesteel, et al PRL (2005).]

Manipulation of qubits would be carried out by moving quasiparticles around each other, not bringing them close together. Advantage: exponentially long decoherence times.

Caveat: Moore-Read state is not rich enough for general topological quantum computation).

More complicated non-abelian states have been proposed, which would allow universal quantum computation. (E.g., Read-Rezayi k=3 state; may be realized at $\nu = 12/5$.)

Quasiparticles in the quantized Hall state correspond to vortices in the superconductor.

For a BCS superconductor with pairing function $\Delta \propto (p_x+ip_y)$: Vortices have zero energy states. If there are 2N vortices present, there are $M=2^{N-1}$ degenerate ground states.

Moving vortices around each other generates a unitary transformation on these states similar to that for quasiparticles in the Moore-Read state.

*Effects of vortex motion on zero energy states elucidated by Ivanov (PRL 2001); and Stern, von Oppen and Mariani (PRB 2004), using Bogoliubov-de Gennes equations for superconductor.

Zero-energy modes

Specifically, in a p_x+ip_y superconductor. an isolated vortex, at point $\mathbf{R}_i$, has a zero energy mode, with Majorana fermion operator γ_i : (from solution of the Bogoliubov-de Gennes equations)

$$\gamma_i = \gamma_i^\dagger \ , \quad \gamma_i^2 = 1 \ , \quad \{\gamma_i \ , \gamma_j\} = 2\delta_{ij}$$

To form ordinary fermion creation or annihilation operator: need pair of vortices: e.g.

$$c_{12} = (\gamma_1 + i\,\gamma_2)/2 \ , \qquad c_{12}^\dagger = (\gamma_1 - i\,\gamma_2)/2 \ ,$$

obey usual fermion commutations rules

$N_{12} = c_{12}^\dagger c_{12}$ has eigenvalues = 0, 1. $[N_{12}, N_{34}] = 0$, etc.

Constraint : Number of occupied pairs = $N_{electrons}$ (mod 2) .

-> 2N vortices gives 2^{N-1} independent states

Explicit relation between Majorana operator and electron operators

$$\gamma_i = \int d\mathbf{r}\, [\, u(\mathbf{r})\, \psi(\mathbf{r}) + v(\mathbf{r})\, \psi^\dagger(\mathbf{r})\,]$$

with $v(\mathbf{r}) = u^*(\mathbf{r})$, localized near vortex.

If vortices are far apart, so there is no overlap between the wave functions of their zero-energy states, then these states must have precisely zero energy.

This relates to the fact that solutions of the BdG equations must occur in pairs with $E_1 = -E_2$.

Boundary states

Finite sample with an odd number of elementary vortices will have a zero energy state at the boundary. Form pair between boundary state and one of the vortices. Again have constraint : Number of occupied pairs = $N_{electrons}$ (mod 2) .

Generally, edge of superconductor has a series of low-energy fermion modes, with energies $E_m = m\,(\pi\, v / L)$,

$m = 0, \pm 1, \pm 2$, ... if number of vortices is odd,

$m = \pm 1/2, \pm 3/2$, ... if number of vortices is even.

Get even-odd alternation in energy to add an electron to the system, if number of vortices is even, not if number of vortices is odd. Alternation energy $\sim v / L$; goes to 0, for $L \rightarrow \infty$

Contrast to s-wave superconductor

s-wave superconductor has no low-energy fermion states at boundary.

Energy to add an electron has an even-odd alternation independent of whether there are an even or odd number of vortices present, and independent of the perimeter of the sample.

True also for gapless d-wave superconductor, $\Delta \propto (p_x+ip_y)^{2,}$ or for a Bose condensate of tightly bound p-wave pairs .

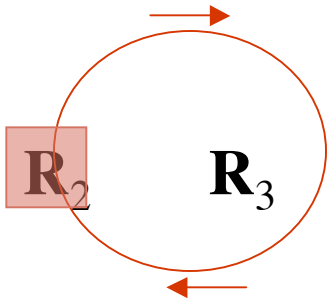
Braiding properties of vortices

Vortices at points $\mathbf{R}_1$ $\mathbf{R}_2$ $\mathbf{R}_3$ $\mathbf{R}_4$

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Braiding properties

Vortices at points $\mathbf{R}_1$ $\mathbf{R}_2$ $\mathbf{R}_3$ $\mathbf{R}_4$

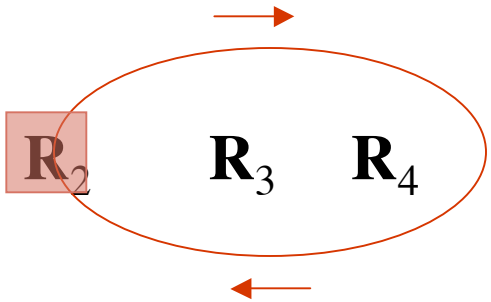


Move vortex 2 around vortex 3 . Gives unitary transformation $\sim \gamma_2 \gamma_3$.

Changes $N_{12} \rightarrow (1-N_{12})$, $N_{34} \rightarrow (1-N_{34})$.

Braiding properties

Vortices at points $\mathbf{R}_1$ $\mathbf{R}_2$ $\mathbf{R}_3$ $\mathbf{R}_4$



Move vortex 2 around 3 and 4. Gives unitary transformation $\sim \gamma_2\gamma_4\ \gamma_2\gamma_3 = \gamma_3\ \gamma_4$: leaves N_{12} and N_{34} unchanged.

Since vortices are indistinguishable, get other unitary transformations by simply interchanging positions of two vortices.

Order of interchanges matter: The unitary transformations do not commute.

Connection between Moore-Read State and p_x+ip_y Superconductor

Moore-Read state is a p_x+ip_y superconductor of "composite fermions"

Connection can be understood through the Fermion-Chern-Simons transformation. Apply a unitary transformation to convert interacting electron system into a set of fermions interacting with a Chern-Simons gauge field (long-range momentum-dependent interaction). Two CS flux quanta are attached to each fermion. In mean field approximation, replace CS field by average field, $\langle b \rangle = 4\pi n_e$ (in units where e=1).

ν=5/2: = one filled Landau level with both spin states occupied + ν=1/2 for electrons in the second Landau level. Assume spin polarized.

If the electron filling fraction is $\nu=1/2$

2 flux quanta of actual magnetic field per electron.

Effective magnetic field = B - <b> = 0.

Mean-field ground state = filled Fermi Sea $k_F = (4 \pi n_e)^{1/2}$

If this is correct, then there is **no energy gap**, **no QHE**.

Should be able to calculate all properties of $\nu=1/2$ state using perturbation theory, starting from the mean field state. Perturbations include effects of $v(\mathbf{r}_i-\mathbf{r}_j)$ and fluctuations in the Chern Simons field $\Delta b_i \equiv b_i - \langle b \rangle$.

Depending on the short-distance interactions between fermions the Fermi surface may be unstable, e.g., to

formation of p-wave superconductivity

If a superconducting energy gap forms at the Fermi surface, then state is stabilized at precisely $\nu = 1/2$. Deviations in filling fraction => $B_{eff} \neq 0$ => requires vortices, costs finite energy.

Get plateau in Hall conductance at $\nu = 1/2$: fractional quantized Hall state.

Apparently: Superconductivity does not occur for electrons in the lowest Landau level ($\nu = 1/2$) but does occur for electrons in the second Landau level.

Moore-Read quasiparticle <=> vortex in superconductor

By Meissner effect, vortex must bind 1/2 quantum of magnetic flux to have finite energy. With a Chern-Simons gauge field, the source of magnetic flux is charge, rather than current.

1/2 quantum of Chern-Simons flux requires 1/4 electric charge.

What is the evidence that the ν =5/2 Quantized Hall State is indeed of the Moore-Read type ?

Evidence comes primarily from numerical calculations on finite systems.(Morf & collaborators, 2002, 2003; Das Sarma et al. 2004).

Using electron-electron interactions appropriate for electrons in the second Landau level, with parameters appropriate to GaAs samples, find a spin-polarized ground state, which seems to have an energy gap, and which has good overlap with Pfaffian wave function. Relatively small changes in parameters can lead to other ground states, which are not quantized Hall states. (As is found experimentally for samples in large in-plane magnetic field.)

Proposed experiments

Moving one quasiparticle around another can be done in principle by means of gates which couple electrostatically to the charge of the quasiparticles; but we are far from being able to accomplish this technologically.

We seek other experiments to examine the $\nu=5/2$ state to see if it is of the Moore-Read type.

*Measurements of the quasiparticle charge. (e.g. using SETs, as in studies of $\nu = 1/3$ by Yacoby et al.) Moore-Read quasiparticles have charge e/4 .

*Measurements of spin polarization. Moore-Read has complete polarization in second Landau Level.

***Interference-type experiments related to non-abelian statistics.**

Proposed Interference Experiments

Discussed by: Ady Stern and B. I. Halperin

Phys. Rev Letters (2006)

Other theoretical papers discussing interference experiments with non-abelian quasiparticles include:

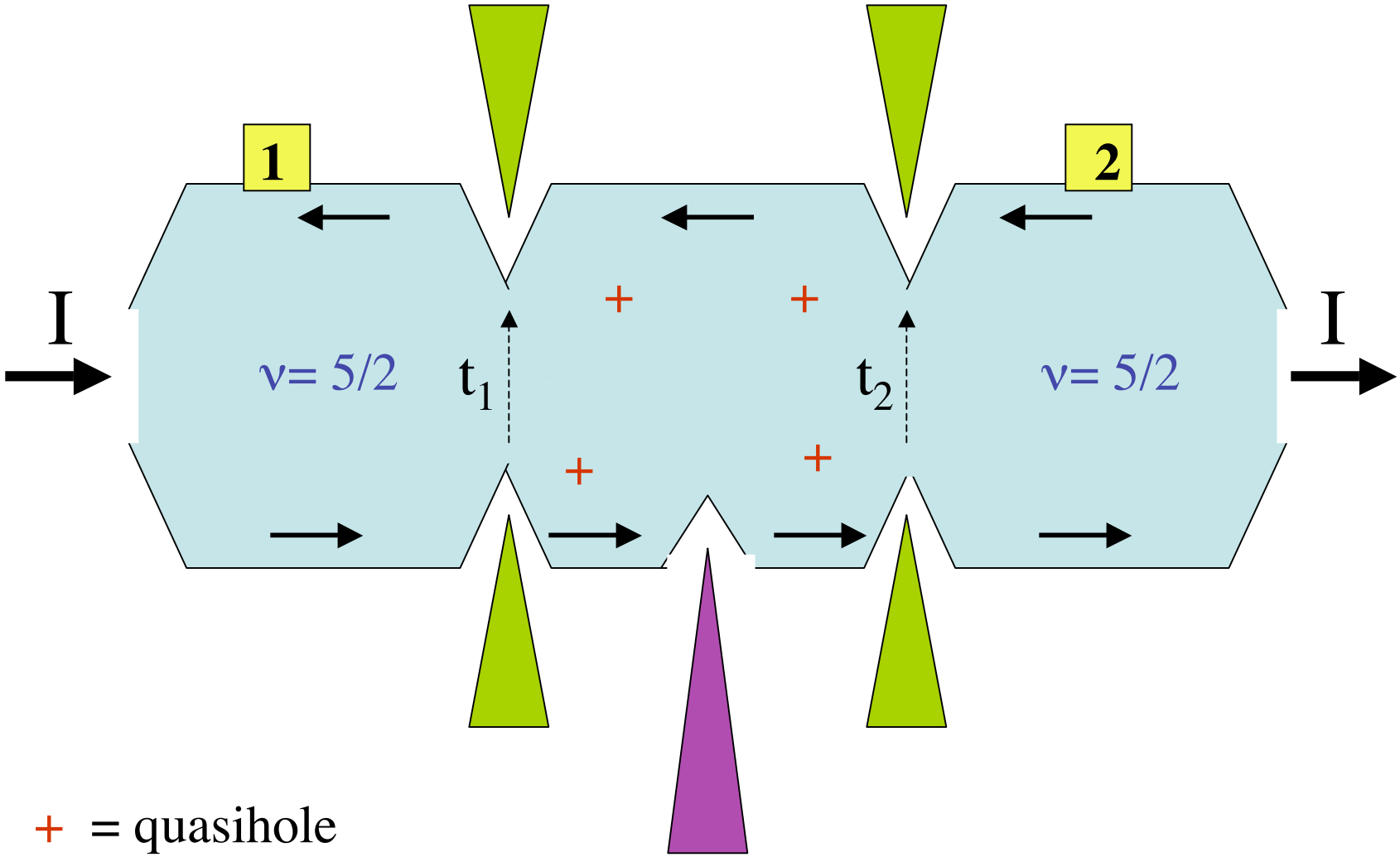
Das Sarma, Freedman and Nayak, (PRL 2005)

Bonderson, Kitaev, and Shtengel, (PRL 2006)

Fradkin et al., Nucl Phys B 1998

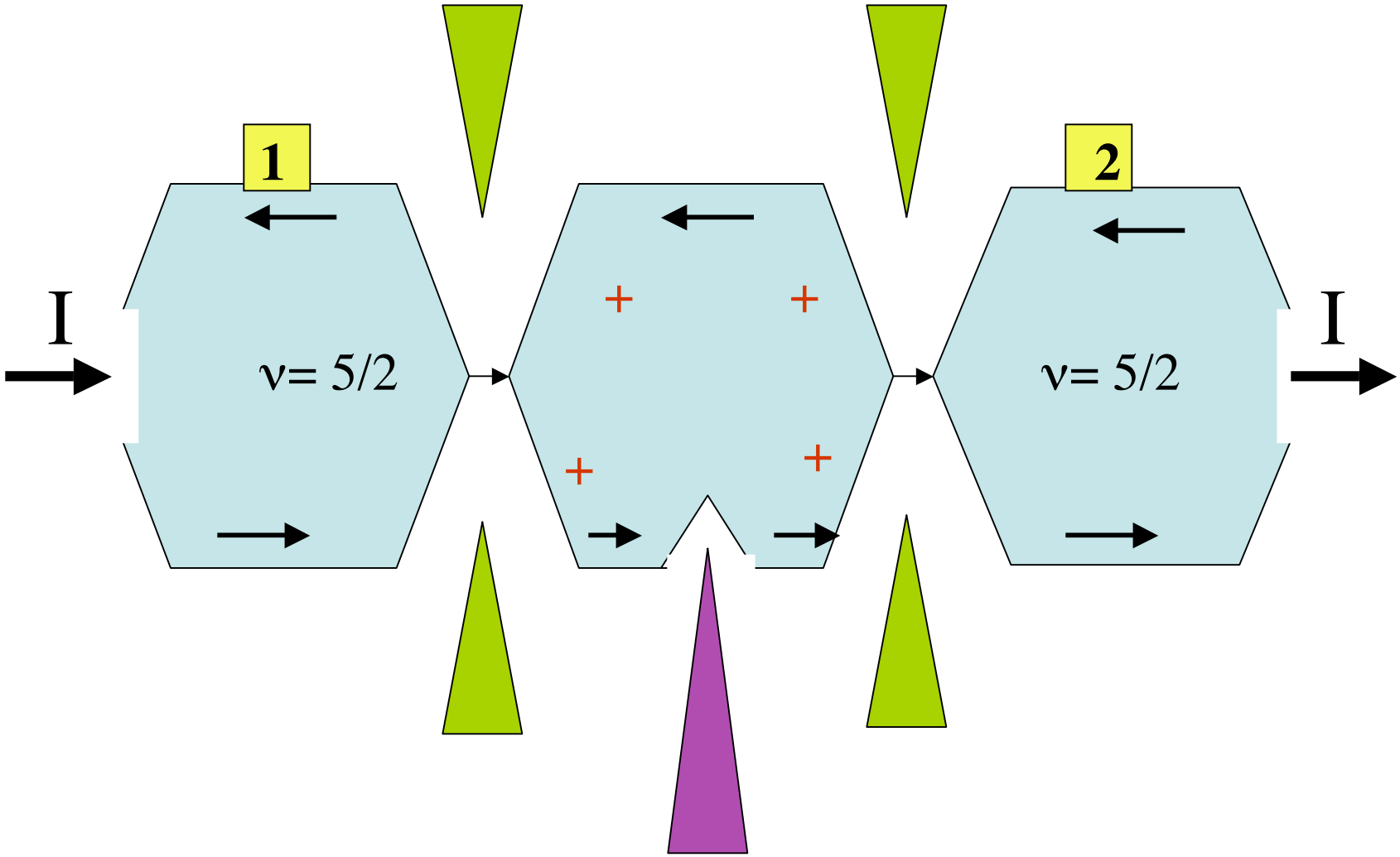
Bonderson, Shtengel and Slingerland, cond-mat/0601242: Discuss consequences for Read-Rezayi parafermion states, possibly applicable to $\nu=12/5$.

Fix gate voltage at point contacts. Vary area A by varying voltage on side gate. Measure resistance V_{12}/I. Expect oscillations in the resistance as a function of A



If $\nu=1/2$ sate is non-abelian Pffafian state: the period of resistance oscillations (for varying A) should depend on whether quasihole-number is even or odd.

Coulomb blockade regime, V_{12} is large, except on resonance, when $E_N=E_{N+1}$



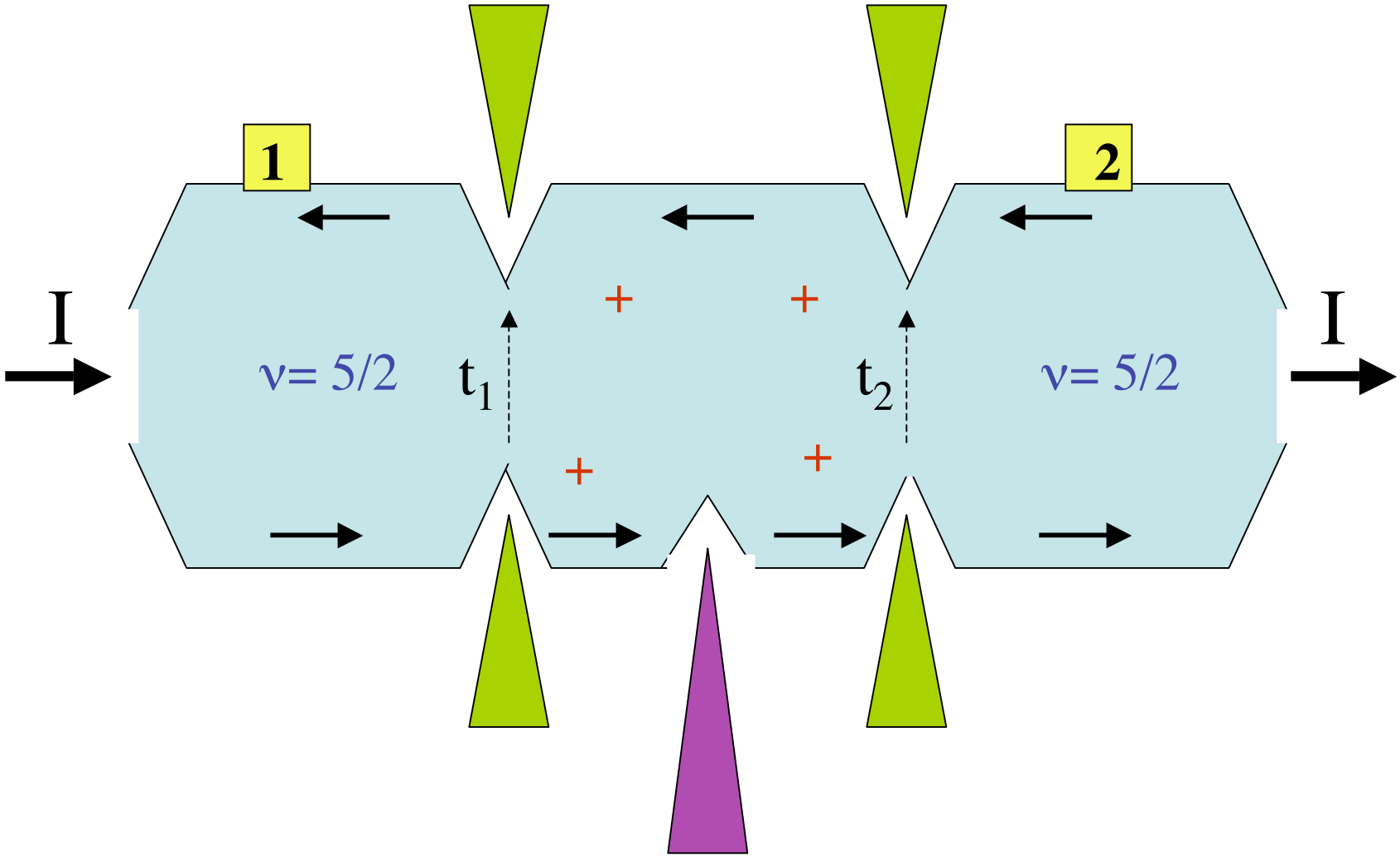
If the number of quasiholes is even, E_N has an even-odd alternation in the electron number N.

The period for oscillations in V_{12} is then $\Delta A = 4\Phi_0/B$, corresponding to the addition of two electrons to the partially-filled landau level, just as for the case of weak back-scattering.

If the number of quasiholes is odd, there is no even-odd alternation in E_N. The period oscillations is then $\Delta A = 2\Phi_0/B$. Again, as in the weak back-scattering case, there is no oscillation with period $\Delta A = 4\Phi_0/B$

Different behavior for even and odd quasihole number is a consequence of the particular non-abelian statistics of the Moore-Read state.

Weak back-scattering: $V_{12} \propto \left| t_1 + t_2\, e^{2\pi i \Omega} \right|^2$, with $\delta\Omega = \delta A\, B/4\Phi_0$, if the qh number is even.



If central region contains an odd number of localized quasiparticles, this interference term is absent. Then leading interference term varies as

Re $[t_1^* \, t_2 \, e^{2\pi i \Omega}]^2$. (Period corresponds to an area containing two flux quanta, rather than four.)

Conclusion

Number parity is a subtle and fascinating issue in fractional quantized Hall states, as well as in superconductors.

There are still many aspects of this subject which are poorly understood.

I hope Vinay may find some time to think about these questions, now that he is “retired.”

The Power of the Sun



Nobel Laureate Walter Kohn will present a new documentary on solar electricity, created in collaboration with Nobel Laureate Alan Heeger and host/narrator John Cleese.

The film is a scientific morality tale: how, starting from the most pure and basic science, through stages of brilliant applied science and engineering, there emerges a promising multi-billion dollar technology to help deal with one of the great challenges of our time: energy. That is, finding economically realistic, clean and safe energy sources to replace diminishing cheap fossil fuels at a time when energy demands of the developing world continue to grow rapidly.

Free and Open to the Public
Saturday, June 16, 2007
4:35pm
Schwartz Auditorium
Rockefeller Hall

Symposium remarks about P-G de Gennes

Thank you, distinguished guests, for voyaging to Ithaca, not the easiest odyssey, for this event! I consider it a great personal compliment. Thank you also to the local participants for sacrificing a typically beautiful Ithaca Saturday to attend.

My purpose now, however, is to remember someone who is not here. As most of you know, Pierre-Gilles de Gennes died on May 18. He had intended to be a speaker, and for most of the year was listed on the web site of this symposium, followed by the words (health permitting). Communications from him during the year had preprints on diverse topics attached to them. In mid-April he sent what has now to be viewed as a poignantly wishful acceptance, and the title: "High Tc superconductors: what makes copper special." On May 11, I received an email from him saying that he was too unwell to travel. A week later he was gone.

This is not the time or place for an extended eulogy. All of you knew him as an astonishingly versatile and inventive physicist; some of us as a colleague and friend. He had enough self-confidence to make lesser talents feel that they had something important to contribute to our shared enterprise. I, personally, am saddened and shaken by his all-too-early demise. Shall we stand in silence for a few moments to remember a great and creative person?

Thank you. [I will send a note to his widow informing her of this act.]

Vinay Ambegaokar
June 16, 2007

Symposium Attendees

Tomás Arias, Cornell University
Alexis Baratoff, University of Basel
Chava Brender, Bar-Ilan University
Sudip Chakravarty, University of California, Los Angeles
Woo Song Choi, Cornell University
Aashish Clerk,* McGill University
Theja De Silva, Cornell University
Ulrich Eckern, University of Augsberg
Veit Elser, Cornell University
Grigory Eremeev, Cornell University
Carl Franck, Cornell University
Carl Frederick, CRG
Paul Ginsparg, Cornell University
Daniel Goldbaum, Cornell University
Robert J. Gooding, Queens University
Kurt Gottfried, Cornell University
Allan Griffin,* University of Toronto
Bertrand Halperin, Harvard University
Louis Hand, Cornell University
Chris Henley, Cornell University
Selman Hershfield, University of Florida
Tin-Lun Ho, The Ohio State University
Donald Holcomb, Cornell University
Eugene Kalinin, Cornell University
Tom Kinoshita, Cornell University
Walter Kohn, University of California, Santa Barbara
James Langer, University of California, Santa Barbara
Andre Leclair, Cornell University
David M. Lee, Cornell University
Anthony Leggett, University of Illinois
Miguel Levy, Michigan Technological University
Johannes Lischner, Cornell University
Alexei Maradudin, University of California, Irvine
N. David Mermin, Cornell University
Erich Mueller, Cornell University
Hiro Miyake, Cornell University
Bruce Patton, The Ohio State University
Mohit Randeria, The Ohio State University
Hassan Raza, Cornell University
John Reppy, Cornell University
Gerd Schon, University of Karlsruhe
Joseph Serene,* Georgetown University
James Sethna, Cornell University
Eric Siggia, Rockefeller Univeristy and Cornell University
Robert Silsbee, Cornell University
Ravindra Sudan, Cornell University
Stephen Teitel, University of Rochester
André-Marie Tremblay, University of Sherbrooke
Ari Turner, Harvard University
Leaf Turner, Cornell University
Cyrus Umrigar, Cornell University
Maxim Vavilov, University of Wisconsin–Madison
Watt Webb, Cornell University
Normal Witriol, CRG
Jan von Delft, University of Munich
Xiaoliang Zhu, Wells College and Cornell University

*Organizing Committee



Attendees

50 Years of Condensed Matter Physics:
A Symposium on the Occasion of Vinay Ambegaokar's Retirement

Saturday, June 16, 2007
Schwartz Auditorium, Rockefeller Hall
Cornell University
Ithaca, NY 14853-2501